

EXERCISES IN GENERAL EQUILIBRIUM MODELING USING GAMS

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TABLE OF CONTENTS

INTRODUCTION AND OVERVIEW	3
EXERCISE 1: A SIMPLE WALRASIAN MODEL	6
INTRODUCTION	6
VERBAL MODEL PRESENTATION	6
MATHEMATICAL MODEL STATEMENT	7
DATA BASE	9
TASK	10
HINTS AND SUGGESTIONS	10
APPENDIX: EXERCISE A1	14
INTRODUCTION	14
TASK	14
HINTS	14
EXERCISE 2: INTRODUCING INTERMEDIATE DEMAND	16
INTRODUCTION	16
DATA BASE	16
TASKS	16
HINTS AND SUGGESTIONS	17
EXERCISE 3: INTRODUCING THE SAVINGS-INVESTMENT BALANCE AND ACTIVITY SPECIFIC WAGES	18
INTRODUCTION	18
EXERCISE 3A: INTRODUCING THE SAVINGS-INVESTMENT BALANCE	18
INTRODUCTION	18
DATA BASE	18
TASKS	19
HINTS AND SUGGESTIONS	20
EXERCISE 3B: INTRODUCING ACTIVITY SPECIFIC WAGES AND UNEMPLOYMENT	21
INTRODUCTION	21
DATA BASE	21
TASKS	21
HINTS AND SUGGESTIONS	22
EXERCISE 4: INTRODUCING GOVERNMENT	24
INTRODUCTION	24
DATA BASE	24
TASKS	25
HINTS AND SUGGESTIONS	26
EXERCISE 5: INTRODUCING THE REST OF THE WORLD	29
INTRODUCTION	29
DATABASE	29
TASKS	42
HINTS AND SUGGESTIONS	42
REFERENCES	44

INTRODUCTION AND OVERVIEW²

Computable General Equilibrium (CGE) models are a class of economy-wide models that are widely used for policy analysis. The term "computable" refers to the fact that the model solution can be computed, a prerequisite when a model is used for applied purposes. A general equilibrium model explicitly recognizes that an exogenous change (in policy or from some other source, for example world markets) that has an impact on any one part of the economy can give rise to repercussions throughout the system. The use of a CGE model is preferable when these repercussions are essential in assessing the impact of these shocks. Mathematically, a standard CGE model consists of a set of simultaneous nonlinear equations. Economically, its starting point is Walras' neoclassical world. However, CGE models used for applied policy analysis, including food policy, tend to deviate considerably from this starting point, incorporating a relatively large amount of detailed real-world structure. CGE models have been used to analyze the impact of policy shifts and external shocks in a wide range of contexts where an understanding of the economy-wide repercussions of the change is required.

This paper presents a revised version of a set of exercises initially developed for use in a Master's level course in CGE modeling taught by the author while at the American University in Cairo. Earlier versions have also been used in the graduate program in Economics at George Washington University, Washington, D.C. and in a training program for researchers at l'Institut d'Economie Quantitative, Ministère du Développement Economique, Tunis. The materials are also appropriate for advanced undergraduate courses.

The purpose of the exercises is to develop the ability of the reader to construct, modify and conduct simulations with CGE models in the GAMS language, the format of which is closely linked to standard mathematical notation.³ This manual comes with a CD ROM with a limited-

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³ GAMS (the General Algebraic Modeling System) consists of a language compiler and integrated solvers. GAMS has strong data processing facilities and can be used to solve a wide variety of optimization and simultaneous equation models. It is one of the most popular softwares for solving CGE models. The web site of the GAMS Development Corporation (www.gams.com) provides information on how to acquire the full-capacity versions of the software as well as access to a wide range of GAMS-related resources, including the manual (Brooke, Kendrick, Meeraus, and Raman, 1998).

capacity version of GAMS, including solvers for linear, non-linear, and mixed-complementarity problems. The approximate background requirements are computer literacy, basic familiarity with GAMS, knowledge of macro and micro theory at the intermediate level, and basic mathematics for economists (including the ability to derive first-order conditions for constrained optimization problems). To carry out these exercises, you need a personal computer of the IBM-type and the GAMS software. All the software now comes with GAMS Integrated Development Environment (GAMSIDE), a windows interface that most GAMS modellers now use.⁴ These exercises should be studied in conjunction with other materials on general equilibrium modeling and relevant economic theory.⁵

The content of the Exercises is outlined in Table 1. The approach is to start with a very simple model and subsequently modify it step by step. With very few exceptions, the models of this volume are based on data presented in the form of Social Accounting Matrices (SAMs). In Exercise 1, the mathematical statement of the simple model and its SAM are provided; the reader's task is to implement the model in GAMS. In the model statements for these Exercises (as well as most models in the CGE literature), the model elements (equations, parameters, and variables) are defined over sets. The Appendix to Exercise 1 includes Exercise A1 in which part of the model of Exercise 1 is rewritten in "longhand," i.e., without any references to sets; the purpose is to remind the reader of what is hidden behind references to sets and elements in model statements.

Starting from Exercise 2, cumulative modifications are introduced into the model of Exercise 1. The model of the final Exercise 5, which includes a critical minimum of real-world features, provides a starting point for more detailed country-specific models that can be used for policy analysis.

⁴ Some older modellers prefer to run GAMS in a DOS environment, which can be done using any text editor that can generate ASCII files. The Microsoft MS-DOS Editor, included with the MS-DOS and Microsoft Windows operating systems is fully adequate. Alternatively, modelers may use the Windows based Integrated Development Environment (IDE) as a substitute.??

⁵ Informative surveys of CGE models include Decaluwe and Martens(1988), Robinson (1989), and Bandara (1991). References to and examples of CGE-based analyses of food policy in developing countries done at IFPRI's Trade and Macroeconomics Division are found in the relevant section of IFPRI's website (www.cgiar.org/ifpri). Extensive treatments of CGE methods are found in Dervis, de Melo and Robinson (1982), Shoven and Whalley (1992), Dixon, Parmenter and Wilcoxon (1992), and Ginsburgh and Keyzer (1997). Condon, Dahl and Devarajan (1987), and Devarajan, Lewis and Robinson (1994) focus on the implementation of CGE models in GAMS.

Table 1. Outline of Exercise content

Exercise #	Content/New feature
1	Simple CGE model
<i>AI</i>	<i>Optional: Simple CGE model in "longhand"</i>
2	Intermediate demands
3a	Savings and investments;
3b	Activity-specific wages; labor unemployed
4	Government
5	Rest of World (open economy)

In Exercises 2-4, the student is provided a verbal description of the model change (with various hints) and a new SAM, and any supplementary data (if needed), on the basis of which he or she is asked to first present a new mathematical statement and, subsequently, implement the modified model in GAMS. For the more complex Exercise 5, the task is limited to implementing the new model in GAMS; the new mathematical statement is provided. The model of the final Exercise 5 includes a critical minimum of real-world features. Hence, it provides a starting point for more detailed country-specific models that can be used for policy analysis. The numbers in the SAMs are fictitious.

CGE modeling is not a spectator sport. It is the hope of the author that, by working through these Exercises, the reader can take an important step toward using CGE models as a tool for policy analysis.

EXERCISE 1: A SIMPLE WALRASIAN MODEL

INTRODUCTION

This exercise involves implementing a simple CGE model in GAMS. The model is presented below, in both verbal terms and the form of a mathematical statement. The model presentation is followed by a SAM which includes the data needed to solve the model using "calibration": on the basis of a data set for a base period, the parameters of the model are estimated in a manner which enables the model (general equilibrium) solution to precisely replicate the base-year data set. Behavioral parameters are calibrated as if the base-year economy was indeed in equilibrium. The functional forms for the various relationships embodied in this Exercise have been selected so as to assure that all parameters can be derived from the accompanying SAM. (With a few exceptions, this is also true for the rest of the exercises in this manual.)

VERBAL MODEL PRESENTATION

We assume that producers maximize profits subject to production functions with primary factors as arguments while households maximize utility subject to budget constraints. Cobb-Douglas functions are used for both producer technology and the utility functions from which household consumption demands are derived. Factors are mobile across activities, available in fixed supplies, and demanded by the producers at market-clearing prices (rents). On the basis of fixed shares (derived from base-year data), factor incomes are passed on in their entirety to the households, providing them with their only income. The outputs are demanded by the households at market-clearing prices.

The model satisfies Walras' law in that the set of commodity market equilibrium conditions is functionally dependent. Any one of these conditions can be dropped; in the proposed model, we drop the equilibrium condition for the non-agricultural commodity. The model is homogeneous of degree zero in prices; to assure that only one solution exists, a price normalization equation, in this case fixing the consumer price index (CPI), has been added. After these adjustments, the model has an equal number of endogenous variables and independent equations. Given this definition of the price normalization equation, all simulated price changes can be directly interpreted as changes vis-a-vis the CPI. The model is disaggregated into two

households (urban and rural), two factors (labor and capital), and two activities and associated commodities (agriculture and non-agriculture). The explicit distinction between activities and commodities facilitates model calibration but is not necessary for the CGE models of this volume. It is, however, needed for models that deviate from a one-to-one mapping between activities and commodities, i.e., for models where at least one activity produces more than one commodity and/or at least one commodity is produced by more than one activity. The label "simple" is well deserved since the model does not include a government, intermediate demands, savings, investment, or an outside world.

MATHEMATICAL MODEL STATEMENT

The mathematical statements and the GAMS input files that accompany this volume follow the current standard notation used in CGE models developed at IFPRI's Trade and Macroeconomic Division. All endogenous variables are written in uppercase Latin letters whereas parameters (including variables with fixed or exogenous values) have lower-case Latin or Greek letters. Subscripts refer to set indices, with one letter per index. Superscripts are part of the parameter name (i.e., not an index). In terms of letter choices, variables and parameters for commodity and factor quantities start with the letter q ; for commodity and factor prices, the first letters are p and w , respectively.⁶

Sets

$f \in F$ factors

{LAB labor
CAP capital}

$h \in H$ households

{U-HHD urban household
R-HHD rural household}

$a \in A$ activities

{AGR-A agricultural activity
NAGR-A non-agricultural activity}

$c \in C$ commodities

{AGR-C agricultural commodity
NAGR-C non-agricultural commodity}

⁶For a discussion of style in economic modeling, see Kendrick (1984).

Parameters

ad_a	efficiency parameter in the production function for activity a
cpi	consumer price index (CPI)
$cwts_c$	weight of commodity c in the CPI
$shry_{h,f}$	share for household h in the income of factor f
qfs_f	supply of factor f
$\alpha_{f,a}$	share of value-added for factor f in activity a
$\beta_{c,h}$	share in household h consumption spending on commodity c
θ_{ac}	yield of output c per unit of activity a

Variables

P_c	market price of commodity c
PA_a	price of activity a
Q_c	output level in commodity c
QA_a	level of activity a
QF_{fa}	demand for factor f from activity a
$QH_{c,h}$	consumption of commodity c by household h
WF_f	price of factor f
$YF_{h,f}$	income of household h from factor f
YH_h	income of household h

Equations

Production and Commodity Block

Activity production function

$$QA_a = ad_a \prod_{f \in F} QF_{fa}^{\alpha_{fa}} \quad a \in A \quad (1)$$

Factor demand

$$WF_f = \frac{\alpha_{fa} \cdot PA_a \cdot QA_a}{QF_{fa}} \quad f \in F, a \in A \quad (2)$$

Activity price

$$PA_a = \sum_{c \in C} \theta_{ac} \cdot P_c \quad a \in A \quad (3)$$

Commodity output

$$Q_c = \sum_{a \in A} \theta_{ac} \cdot QA_a \quad c \in C \quad (4)$$

Institution Block

Factor income

$$YF_{hf} = shry_{hf} \cdot WF_f \cdot \sum_{a \in A} QF_{fa} \quad h \in H, \quad f \in F \quad (5)$$

Household income

$$YH_h = \sum_{f \in F} YF_{hf} \quad h \in H \quad (6)$$

Household demand

$$QH_{ch} = \frac{\beta_{ch} \cdot YH_h}{P_c} \quad c \in C, \quad h \in H \quad (7)$$

System constraint block

Factor market equilibrium

$$\sum_{a \in A} QF_{fa} = qfs_f \quad f \in F \quad (8)$$

Output market equilibrium

$$Q_c = \sum_{h \in H} QH_{ch} \quad c \in C \quad (9)$$

Price normalization equation

$$\sum_{c \in C} cwtsc \cdot P_c = cpi \quad c \in C \quad (10)$$

DATA BASE

The data base of the model is presented in Table 1.1.

Table 1.1: Social Accounting Matrix for Exercise 1

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	Total
AGR-A			125						125
NAGR-A				150					150
AGR-C							50	75	125
NAGR-C							100	50	150
LAB	62	55							117
CAP	63	95							158
U-HHD					60	90			150
R-HHD					57	68			125
Total	125	150	125	150	117	158	150	125	

TASK

Using GAMSIDE, input the above model using GAMS syntax. Solve the model in GAMS and verify that the solution can replicate the above SAM.

HINTS AND SUGGESTIONS

1. Before attempting to do this Exercise, the reader should be familiar with GAMS, at least at the level of the tutorial Chapter 2 in the User's Guide (Brooke, Kendrick, Meeraus, and Raman, 1998, pp. 5-28). The rest of this guide is an indispensable reference (cf. Footnote 3).
2. You may run the input file in GAMS at any point in the process of constructing the model. If the model is incomplete and hence not solved, GAMS will nevertheless check that the input conforms with its syntax, report any errors, and, in the absence of errors, carry out other instructions, including displays. To catch errors at an early stage, it is often helpful to inspect the results of displays of elements (sets, variables and parameters), that have been defined via operations.
3. A timesaving device when developing the model is to use mechanical searches for text segments, including “****”, used to indicate errors in the output file of GAMS (which by default is called *myfile.lst* if the input file is called *myfile* or *myfile.gms*).
4. The above mathematical statement is provided in a format that can be easily implemented in GAMS. It is advisable to use the same notation (subject to various minor transformations) since this will save effort (no additional notation is needed) and make it easier to move between the mathematics and GAMS.⁷ For example, the first equation, with PRODFN as its declared name, may be input as follows:

PRODFN(A).. QA(A) =E= ad(A)*PROD(F, QF(F,A)**alpha(F,A));

5. In order to facilitate SAM-related computations, it is helpful to generate a global set,

⁷ Note that GAMS is case-insensitive. Nevertheless, it is probably easier to read a GAMS statement where the distinction between variables and parameters is evident, for example with parameters in lower case and variables in upper case (the proposed convention).

here named AC, including all elements in the sets for factors, activities, commodities, and households. Sets for the latter items are subsequently declared and defined as subsets of the global set.

6. When building CGE models, it is often useful to have identical sets with different names. In GAMS, the ALIAS command may be used to create a set that is identical to a set that already has been defined. (In the suggested answer, ALIAS is used to define sets identical to AC, C and F.)
7. The models in this volume are formulated as a set of simultaneous equations and solved using a solver for nonlinear mixed-complementarity problems (PATH or MILES).⁸ Alternatively, simultaneous-equation models may be solved as nonlinear optimization problems. To follow this approach, you may define an additional equation, OBJFN, as:

OBJFN.. OBJ =E= DUMMY**2;

where OBJ is an unconstrained variable, and DUMMY is a nonnegative variable. The solution statement should be changed to SOLVE CGE1 USING NLP MINIMIZING OBJ. In this setting, the other equations would define the constraint set with only one feasible solution (identical to the solution to the corresponding simultaneous-equation model). OBJ is minimized when DUMMY has a value of zero.

8. Do not input the row and column totals of the SAM. Instead, to remove one source of errors and model infeasibility, compute row and column totals and check that they are identical.
9. After having input the SAM, start using it to define the parameter values and initial variable levels. Initial variable levels are helpful for two reasons. First, it gives the GAMS solver a good starting point, facilitating its search for the solution. Second, it makes it easier to pinpoint reasons for why a model fails to generate the benchmark equilibrium. The latter aspect is discussed further under point 12.
10. Note that parameters (in this model for example ad_a) may be defined using preceding definitions; when this approach is followed, it matters in which order the parameters and initial variable values are defined.
11. Assume that base-year factor and output prices are at unity and assign parameter and variable quantities on this basis. (This amounts to choosing the unit for each real flow so as to assure that its corresponding price is one.)
12. If you have implemented the model correctly, there should be no "significant" discrepancies between left-hand and right-hand sides of the equations when GAMS plugs in your parameter values and initial variable levels. You check this by looking for three asterisks (***) in the "equation listing." If the left-hand side value (indicated

⁸ In the general case, a mixed-complementarity problem consists of a set of simultaneous equations that are a mixture of strict equalities and inequalities, with the latter linked to bounded variables. The current model is a special case since all equations are strict equalities. For details and a mathematical definition, see Rutherford (1995).

as “LHS = <value>”) is “significantly” different (let’s say by more than 1.0E-5) from the right-hand side value in the preceding equation (after =E=), there is some problem associated with the definitions of the parameters and variables which appear in the equation in question. Errors may be caught by displaying the values of all parameters and variables in any problematic equation and checking whether the values are compatible with the SAM.

13. The number of (single) equations and (single) variables, reported as part of the “model statistics,” should be identical (at 24 for the proposed solution).⁹
14. A value of unity for all factor and commodity prices (that were initialized at this level) is a reliable indicator that the initial model solution replicates the initial equilibrium as captured by the initial SAM. To test that the model is robust, it is a good idea to solve it a couple of times with different values for selected parameters. To check that the model indeed is homogeneous, the initial value of *cpi* may be multiplied by some positive factor; compared to the base, the prices of the model solution should all be changed by the value of the factor while all quantities should stay unchanged.
15. In addition to model statement and base solution, the GAMS input file with the suggested answer includes a LOOP where two simulations are carried out, the BASE and a simulation for which the capital stock is increased by 10%. Report parameters are created to show values for variables and the factor supply parameter (which was changed in the experiment). Variable and parameter values may also be used to generate report parameters that provide additional information not defined by any single model variable, for example national account statistics.¹⁰
16. This first Exercise may be the toughest one; it is certainly the one requiring the largest amount of new GAMS code. It is important that you try to do it *before* checking the suggested answer since “learning by doing” is the name of the game. However, keep in mind that, although the suggested answer tries to embody good modeling practices,

⁹ By default, the GAMS variable count also includes variables that are fixed (have exogenous values) unless the model attribute *holdfixed* has been specified with a value of one, in which case only endogenous variables are included in the count (see Brooke *et al*, 1998, pp. 74-76). If the model includes fixed variables, this attribute makes it straightforward to verify that the number of endogenous variables and equations is equal. In this manual, fixed variables and the *holdfixed* attribute appear in the proposed GAMS solutions to Exercises 3-5.

¹⁰ For a relatively comprehensive set of report parameters for a CGE model, see the GAMS listing of the Cameroon CGE model in Chapter 3 of Devarajan *et al.* (1994).

it is merely a *suggested* answer. Different formulations may seem preferable to other users. In many ways, "style" in modeling and language involve similar considerations, including room for taste differences.

APPENDIX: EXERCISE A1

INTRODUCTION

The notation of the model of Exercise 1 is set-driven, *i.e.*, reference is frequently made to sets and set indices.¹¹ The purpose of this Exercise is to provide practice in interpreting what is hidden behind the veil of set notation and to demonstrate the gains from set notation in the form of more concise and more easily modified models. For example, the disaggregation of models based on set notation can be changed simply by varying the set definitions, the SAM and other data, without any other changes in the input file.¹² The starting point for this Exercise is the GAMS input file with your correct answer to Exercise 1.

TASK

In the GAMS statement, rewrite two of the equations in Exercise 1, PRODFN and FACDEM, in "longhand," *i.e.*, instead of making reference to sets and indices, refer to specific set elements by name. Leave the rest of the model statement unchanged. Run the model and check that the solution is identical to Exercise 1.

HINTS

1. You have to declare and define a total of six equations. (In the suggested answer, they are named PRODFN1, PRODFN2, FACDEM1, FACDEM2, FACDEM3, and FACDEM4.) Put asterisks in the first character position of the lines where reference is made to the equations PRODFN and FACDEM (their definitions and declarations).¹³
2. Write the production function for the agricultural activity as follows:

¹¹ Recall that subscripts (but not superscripts) are set indices.

¹²If one more production activity and commodity is added, the complete current model in long-hand would require six new equations and modifications for five old equations. To add one more activity and commodity in the set-driven statement would merely require the addition of a couple of additional lines in the set definitions. Irrespective of approach, it would be necessary to further disaggregate the SAM.

¹³When an asterisk ("*") is put in the first character position, GAMS ignores the rest of the line but reproduces it in the output file. This is a useful device for including short comments or excluding unused parts of a program without deleting them. (An alternative approach, preferable for longer comments, is to block off a section with \$ONTEXT and \$OFFTEXT before and after the section, respectively. The two "dollar" statements must start in the first character positions of their respectively lines.)

PRODFN1.. QA('AGR-A') =E=
ad('AGR-A')*QF('LAB','AGR-A')**alpha('LAB','AGR-A')
*QF('CAP','AGR-A')**alpha('CAP','AGR-A');

EXERCISE 2: INTRODUCING INTERMEDIATE DEMAND

INTRODUCTION

In this Exercise, intermediate demands are added to the CGE model presented in Exercise 1.

DATA BASE

The new SAM is shown in Table 2.1. New payment flows, representing payments for intermediate goods, have been added in the cells at commodity rows and activity columns

Table 2.1: Social Accounting Matrix for Exercise 2

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	TOTAL
AGR-A			225						225
NAGR-A				250					250
AGR-C	60	40					50	75	225
NAGR-C	40	60					100	50	250
LAB	62	55							117
CAP	63	95							158
U-HHD					60	90			150
R-HHD					57	68			125
TOTAL	225	250	225	250	117	158	150	125	

intersections. The accounts in the SAM are unchanged.

TASKS

1. Mathematical statement. Modify the mathematical statement so that the model incorporates intermediate demands. For both sectors, assume Leontief technology, *i.e.*, that a fixed input quantity is needed per unit of output.

2. GAMS. After having convinced yourself that the answer to (1) is without errors (compare it to the suggested answer), implement the model in GAMS. This involves all or parts of the following: modifying the SAM; adding/modifying declarations and definitions for sets, parameters, variables (for the latter, the “definition” involves defining the initial levels), and equations; displaying (and checking) the results of new computations; solving the model without errors for the base case and for a simple experiment (the latter to check that the model is robust); and confirming that it replicates the base data. Make sure that for each new element (set, variable, parameter or equation), you go through the same steps as for the corresponding

elements already present in the initial model.

Once the model is calibrated to the SAM, run the same experiment as for Exercise 1.

HINTS AND SUGGESTIONS

1. Mathematical statement

- a. When modifying the statement, check that the changes in the number of variables and equations are equal (so that the number of variables and independent equations remains equal). Compared to the Exercise 1 model, the suggested model has six additional equations and variables, the total number being 30.
- b. The suggested answer includes the following new elements:

Parameters

ica_{ca} quantity of c as intermediate input per unit of output in activity a

Variables

PVA_a value-added (or net) price of activity a

$QINT_{ca}$ quantity of commodity c as intermediate input in activity a

There are no new sets but two new equations, for value-added prices and intermediate demands. Some changes in other equations are also needed.

2. GAMS

- a. Be systematic when you modify the model: for every parameter/variable you declare, make sure you don't forget to include it in the equations, or to define and display its value/initial level. Check that the displayed values coincide with the values you expect.
- b. Note that both PVA_a and P_c cannot have initial (or equilibrium) levels of unity. In the suggested answer, we follow the convention of keeping the initial value of P_c at unity and defining PVA_a at a level corresponding to the share of total output value accruing to the primary factors.
- c. As for Exercise 1, define the parameter ad_a so that the production (or, more precisely, value-added) function for activity a on its own will generate the base level for QA_a .
- d. Among the hints for Exercise 1, pay special attention to point 12 regarding how to track down reasons for why the model fails to replicate the base-year equilibrium.

EXERCISE 3: INTRODUCING THE SAVINGS-INVESTMENT BALANCE AND ACTIVITY SPECIFIC WAGES

INTRODUCTION

Few applied CGE models fall short of explicitly covering savings and investment. In our gradual process of constructing an applied model, we start by adding this aspect, using the CGE model of Exercise 2 as our starting point. Moreover, in the previous models, the wage (price) of each factor was assumed to be uniform across all activities that use it; in other words, every activity paid the average wage. In the real world, wages tend to be "distorted" in the very broad sense that they differ across activities. A treatment that permits this feature (with no distortions as a special case) is also introduced in this exercise. We will assume that wages are distorted for labor but uniform across activities for capital in a setting with full (or fixed) employment for both factors.

When doing the Exercise, follow one general aspect of good modeling practice: introduce changes in different areas one at a time. Consequently, we split this exercise in two and will introduce savings and investment first, followed by activity specific wages.

EXERCISE 3A: INTRODUCING THE SAVINGS-INVESTMENT BALANCE

INTRODUCTION

Few applied CGE models fall short of explicitly covering savings and investment. In our gradual process of constructing an applied model, we start by adding this aspect, using the CGE model of Exercise 2 as our starting point.

DATA BASE

The SAM, displayed in Table 3.1, includes one new account called savings-investment (S-I).¹⁴ Its row receives payments from the household (the only saver in this simple economy); its column shows spending on commodities used for investment. Note that investment is defined

¹⁴The change in the assumption about the capital market is not linked to any change in the SAM since the SAM merely reports payment flows, *i.e.*, the SAM does not say anything about the behavioral rules of the

in terms of the commodities used in the production of the capital stock, not the activity of destination (the activity that receives the investment goods as an addition to its capital stock). This means that the model only applies to a period so short that there is not enough time for new investments to provide additional production capacity. For a model relevant to a longer time period (for example a multi-period model), it would also be necessary to explicitly consider the resulting changes in capital stock.

Table 3.1: Social accounting matrix for Exercise 3

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	S-I	TOTAL
AGR-A			250							250
NAGR-A				305						305
AGR-C	60	40					50	75	25	250
NAGR-C	40	60					100	50	55	305
LAB	72	80								152
CAP	78	125								203
U-HHD					80	120				150
R-HHD					72	83				155
S-I							50	30		80
TOTAL	250	305	250	305	152	203	150	155	80	

TASKS

1. Mathematical statement.

Expand the mathematical statement so that savings-investment is included. For savings-investment, assume the following:

- household income is allocated in fixed shares to savings and consumption;
- investment is savings-driven, *i.e.*, the value of total investment spending is determined by the value of savings

investment spending is allocated to the two commodities in a manner such that the ratio between the quantities is fixed.

Together, assumptions (b) and (c) mean that when savings values and/or the prices of investment commodities change, there is a proportional adjustment in the quantities of

investment demand for each commodity, generating an investment value equal to the savings value.

The set of equilibrium conditions that is functionally dependent now includes, the savings-investment balance. It would be possible to drop one of these equations. In the suggested answer, another approach is selected. Instead of dropping one of these equations, a variable called WALRAS is introduced in the savings-investment balance. This approach is commonly followed in this class of models. The model still has an equal number of variables and equations. If the model works, the savings-investment balance should hold, i.e., the value of WALRAS should be zero.

2. GAMS

After having produced an error-free mathematical statement, implement the model in GAMS, using the same systematic approach that was developed for Exercise 2. Proceed with the introduction of savings-investment. When the base solution works well also with this change, solve for the experiment with a 10% increase in the capital stock. Verify that the value for WALRAS remains (very close to) zero also for this solution.

HINTS AND SUGGESTIONS

1. Mathematical statement.

- a. Introduce the changes in a step-wise manner, in each step keeping track of changes in the number of variables and equations.
- b. For the savings-investment modification, the following changes may accomplish the task:

Parameters: Introduce new parameters for household savings shares (called mps_h) and base-year sectoral investment quantities (called $\overline{qinv_c}$);

Variables: Add new variables for quantities of investment demand and a factor introducing proportional changes in investment quantities (referred to as $QINV_c$ and $IADJ$, respectively).

Equations: Include new equations to define $QINV_c$ and to impose balance between savings and investment values. The investment equation may be written as $QINV_c = \overline{qinv_c} \cdot IADJ$.

2. GAMS.

Before introducing savings and investment, add S-I to elements in the set AC and use the new SAM.

The suggested GAMS model has 34 equations and variables.

EXERCISE 3B: INTRODUCING ACTIVITY SPECIFIC WAGES AND UNEMPLOYMENT

INTRODUCTION

In the previous models, the wage (price) of each factor was assumed to be uniform across all activities that use it; in other words, every activity paid the average wage. In the real world, wages tend to be "distorted" in the very broad sense that they differ across activities. A treatment that permits this feature (with no distortions as a special case) is introduced in this exercise. We will assume that wages are distorted for labor but uniform across activities for capital in a setting with full (or fixed) employment for both factors.

In addition, we consider a model set-up with a flexible market-clearing wage by assuming that labor is unemployed with fixed, activity-specific real wages and the quantity of labor supply as the market-clearing variable.

DATA BASE

For labor, the number of workers employed is 100 for agriculture and 50 for non-agriculture. For capital, we continue to assign quantities on the basis of the assumption that the wage is unity for both activities. There are no changes in the SAM associated with the change in the factor treatment.

TASKS

1. Mathematical statement.

Expand the mathematical statement so that each activity pays fixed shares of average base wages for labor and capital. For the factor markets (both labor and capital), assume that each activity pays an endogenous wage expressed as the product of an endogenous (economywide) wage variable (for the base equal to the average wage) and an exogenous distortion factor. For

the special case of no distortions, the distortion factor is equal to one for all activities. In each factor market, variations in the average wage clear the market.

2. GAMS

After having produced an error-free mathematical statement, implement the model in GAMS, using the same systematic approach that was developed for Exercise 3A. Proceed with the introduction of activity specific wages. When the base solution works well also with this change, solve for the experiment with a 10% increase in the capital stock. Continue verifying that the value for WALRAS remains (very close to) zero also for this solution.

HINTS AND SUGGESTIONS

1. Mathematical statement.

1. Introduce the changes in a step-wise manner, in each step keeping track of changes in the number of variables and equations.
2. The change in the treatment of factor markets may involve the following:
 - **Parameters:** Define and declare a new distortion factor ($wf_{dist_{fa}}$) that represents the ratio between the wage for factor f in activity a and the average wage for factor f ;
 - **Equations:** To assure that payments for factors are made at distorted wages, multiply the average wage variable (WF_f) by the distortion factor in every equation where the wage variable appears. (The definition of the distortion factor implies that $WF_f \cdot wf_{dist_{fa}}$ indeed defines the wage for factor f in activity a .)
 - Introduce fixed, activity-specific real wages and the quantity of labor supply as the market-clearing variable. A relatively flexible approach is suggested. The changes are:
 - i. The parameters $wf_{dist_{fa}}$ and qfs_f are turned into variables, written as $WFDIST_{fa}$ and QFS_f , respectively.
 - ii. Among the factor wage and quantity variables, the following are fixed:

$$WFDIST_{lab,a}, WF_{lab}, QF_{cap,a} \text{ and } WF_{cap}.$$

This approach is relatively flexible since, by selectively fixing factor wage and quantity variables, it can handle a variety of closure rules (including the one used as default in this exercise).

2. GAMS

For factors quantities and wages, you may go through the following steps:

- On the basis of the information in the data base and stated assumptions, define initial levels for the activity-specific factor demand variable (QF_{fa}) and the factor supply parameter (qfs_f).
- Define initial levels for the average wage variable (WF_f) and activity-specific wages (an auxiliary parameter that only is used to facilitate calibration, in the proposed solution called wfa_{fa}).
- Define the wage distortion parameter ($wfdist_{fa}$) as the ratio between wfa_{fa} and WF_f .
- For each activity-factor combination, verify that the product $WF_f \cdot wfdist_{fa} \cdot QF_{fa}$ equals the SAM payment from the activity to the factor.¹⁵

The suggested GAMS model has 34 equations and variables.

¹⁵The market-clearing economy-wide wage variable was initialized at the level of the average base wage. Generally speaking, it will not coincide with the economy-wide average wage for any experiment unless (1) $wfdist_{fa}$ equals one for all activities and/or (2) there is no change in the employment shares for the different activities. (This is confirmed by the results for Exercise 3a.)

EXERCISE 4: INTRODUCING GOVERNMENT

INTRODUCTION

Up to now, the modeled economy has not included a government, an essential actor in applied policy analysis. In this Exercise, this defect is remedied. The government of the model earns its revenues from income and sales taxes and spends it on consumption and transfers to the households. Government savings is the difference between its revenues and spending.

DATA BASE

The model is built around the SAM shown in Table 4.1. Labor employment quantities are the same as for Exercise 3B (100 for agriculture and 50 for non-agriculture). The introduction of the government is behind the changes in the SAM structure. There are new accounts for the government (GOV) and the two tax types, taxes on incomes (YTAX) and sales (STAX). In the tax rows, income taxes are collected from the household and sales taxes from the commodity accounts (AGR-C and NAGR-C). In the tax columns, this income is passed on to the government. The government column shows that the government uses this revenue to cover the cost of government commodity consumption (payments to AGR-C and NAGR-C), transfers to the households (payments to U-HHD and R-HHD), and (negative) government savings (payment to S-I). Note that, in the rows of the commodity accounts, the demanders buy commodities at market prices; in the columns of the commodity accounts, these payments are split between the sales tax account and the activities (paid for output valued at producer prices).

One important part of government consumption, government payment for the labor services of its administrators and other employees, does not appear explicitly in the SAM. They may be viewed as working for a government service activity that produces a commodity which is purchased by the account for the government (institution). In this SAM, this activity-commodity pair is part of the non-agricultural activity and its commodity. In more disaggregated, real-world SAMs, the government service activity and commodity typically have their own accounts.

Table 4.1: Social Accounting Matrix for Exercise 4

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP
AGR-A			255			
NAGR-A				350		
AGR-C	66	44				
NAGR-C	44	66				
LAB	72	105				
CAP	73	135				
U-HHD					95	125
R-HHD					82	83
GOV						
S-I						
YTAX						
STAX			25	33		
TOTAL	255	350	280	383	177	208

	U-HHD	R-HHD	GOV	S-I	YTAX	STAX	TOTAL
AGR-A							255
NAGR-A							350
AGR-C	55	77	11	27			280
NAGR-C	110	55	47	61			383
LAB							177
CAP							208
U-HHD			25				245
R-HHD			5				170
GOV					25	58	83
S-I	60	33	-5				88
YTAX	20	5					25
STAX							58
TOTAL	245	170	83	88	25	58	

TASKS

1. Mathematical statement.

Present a statement for a model, based on the above SAM, that includes a government. Assume the following:

- The income tax is a fixed share of the gross income of each household. A fixed share of post-tax income is saved and the rest is spent on consumption.

- Sales taxes are fixed shares of (mark-ups on) producer commodity prices.
- The government consumes fixed commodity quantities, paying market prices (including the sales tax). Government transfers to the households are CPI-indexed, *i.e.*, they can simply be fixed in nominal terms — indexation to the CPI is automatic since the CPI level is fixed via the price normalization equation.¹⁶
- Government savings is a residual, assuring balance between government outlays (including savings) and revenues. It is computed as the difference between expenditures (excluding savings) and revenues.

2. GAMS

Once you have produced a correct mathematical statement, implement the model in GAMS. Make sure that the model can replicate the database and solves for an experiment where the quantities of government consumption of each commodity are increased by 20%.

HINTS AND SUGGESTIONS

1. Mathematical statement.

To model the government, go through the following steps:

Sets: A new set, I (with an identical set named I') defines institutions (currently the two households and the government; the rest of the world will be added in Exercise 5). It is referred to in the modeling of transfers between institutions.

Parameters: The new parameters, with suggested notation parenthesized, define government commodity consumption (qg_c), sales and income tax rates (tq_c and ty_h , respectively), and transfers from institution i' to institution i ($tr_{i'}$). The transfer parameter captures transfers from the government to the households; in the equations where it appears, reference is made explicitly to the relevant subset (H) and elements (GOV).¹⁷

¹⁶ If so, the model is, strictly speaking, no longer homogeneous of degree zero in prices. (Why?) If you would like to maintain homogeneity, multiply the government transfer parameters by cpi .

¹⁷ Alternatively, it would have been possible to declare this as a government transfer parameter with only the

Variables: The new variables, with the symbols parenthesized, denote producer prices exclusive of the sales tax (PX_c), government revenue (YG), and government expenditures (EG). The sales tax introduces a wedge between the price received by the producers (PX_c) and that paid by the demanders which includes the sales tax (the old symbol P_c is used to define the latter price). Thus, one important task is to change the variable P_c in the Exercise 3 model to PX_c in the current model whenever reference is made to what the producer receives (and not to what the demander pays).

Equations: New equations are needed to define government revenue and expenditures. Modifications are introduced in the equations for household income (government transfers are a new income source), household consumption demand (owing to the presence of income taxes), commodity market equilibrium (to account for government consumption), and the savings-investment balance (since the government represents a new source of savings).

2. GAMS

To model the government, the following hints may facilitate your task:

1. Augment the set AC with accounts for the government, and the two tax types.
2. Declare and define the new set for institutions.
3. Let the initial values be unity for all prices except PVA_u and P_c (i.e., PX_c is among the prices with an initial value of unity).
4. Calibrate the rate of the sales tax (tq_c) as the ratio between the tax payment and output value excluding the sales tax.

receiving set of institutions, h , in its domain. However, in a more complex model with many paying institutions, this approach would be tedious, requiring the definition of a separate parameter for each paying institution. The advantage of defining it over broadly defined sets of paying and receiving institutions is increased flexibility — one single parameter can handle a wider variety of contexts and fewer changes are needed elsewhere in the model. This will be evident in Exercise 5 where the rest of the world is added to the set of paying institutions, transferring money to both households and the government.

5. Given the values of tq_c and PX_c , you can compute the initial value of P_c . Given these values and the values of ica_{ca} , you can find PVA_a .
6. Given that commodity market prices (P_c) paid by the demanders are no longer unity, it is now necessary to consider this price explicitly when computing values for parameters and variables linked to commodity quantities.
7. Note that the household savings rate (mps_h) should now be computed as the ratio between household savings and household disposable (post-tax) income.

In addition to the 34 variables and equations of the previous exercise (CGE3B), we now have variables and equations for two demander prices, government expenditure and government revenue. The suggested model thus has 38 variables and equations.

EXERCISE 5: INTRODUCING THE REST OF THE WORLD

INTRODUCTION

In this final Exercise, we complete the model by adding the rest of the world (RoW). Interaction with the RoW takes place in the form of imports, exports and transfers. Crucially, for demanders, imports and domestic output sold domestically are assumed to be imperfect substitutes. Similarly, for producers, imperfect transformability is assumed between exports and domestic output sold domestically.¹⁸ Compared to the alternative of perfect substitutability (which, for any given commodity, only permits one-way trade), this treatment tends to generate more realistic responses in domestic prices, production and consumption to changes in international prices. The treatment of factor markets is the same as for Exercise 4.

In combination with an appropriately disaggregated SAM, and data for labor employment and elasticities, the model that is the output of this Exercise may provide the starting point for real-world applied policy analysis. However, it is highly likely that changes are needed to reflect better the structure of the modeled economy. Such changes, may, for example, include the introduction of price controls and other features that invalidate the assumption that flexible prices clear perfectly competitive markets. In addition, available production and consumption elasticities would typically suggest that the Cobb-Douglas functions should be replaced by more flexible (and complex) functional forms.

The task of implementing the model from scratch on the basis of stated assumptions is quite complex. Hence, we will provide, not only the SAM, but also a complete mathematical statement with brief comments on new features.

DATABASE

The data base for the model consists of the SAM found in Table 5.1, unchanged data for labor employment, and trade elasticities: elasticities of substitution between imports and domestic sales (of domestic output), and elasticities of transformation between exports and domestic sales. In the model, a value of 2 is used for both elasticities across the two

¹⁸ Imperfect substitutability and transformability may arise from differences in physical quality, differences in time and place of availability, and from aggregation biases.

commodities.

Table 5.1: Social Accounting Matrix for Exercise 5

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD
AGR-A			279					
NAGR-A				394				
AGR-C	84	55					30	49
NAGR-C	50	99					165	92
LAB	72	105						
CAP	73	135						
U-HHD					95	125		
R-HHD					82	83		
GOV								
S-I							70	40
YTAX							20	5
STAX			10	20				
TAR				39				
ROW				105				
TOTAL	279	369	289	533	177	208	285	186

	GOV	S-I	YTAX	STAX	TAR	ROW	TOTAL
AGR-A							279
NAGR-A							394
AGR-C	13	28				30	289
NAGR-C	67	85					558
LAB							177
CAP							208
U-HHD	25					40	285
R-HHD	5					16	186
GOV			25	30	39	15	109
S-I	-1					4	113
YTAX							25
STAX							30
TAR							39
ROW							105
TOTAL	109	113	25	30	39	105	

The SAM itself includes two new accounts, for the rest of the world (ROW) and for import tariffs (TAR). The row of the ROW account shows that our spending on imported commodities is the only income source of the RoW in its dealings with our country; the column of the same account shows that the receipts of our country from ROW consist of export revenues, and

transfers to the households and the government.¹⁹ The payments from ROW to S-I is foreign savings or the current account deficit, i.e., the difference between our country's current (non-capital) foreign exchange expenditures and earnings.

Mathematical statement

The bulk of this statement consists of the model equations (a total of 31), divided into "blocks" for prices, production and commodities, institutions, and system constraints. Explanatory boxes are provided below each equation. New equations and other changes compared to the model of Exercise 4 are explained. The statement starts with alphabetical lists of sets, parameters, and variables that should serve as a reference as the reader goes through the equations

Sets

$a \in A$	activities
$c \in C$	commodities
$c \in CM (\subset C)$	imported commodities
$c \in CNM (\subset C)$	non-imported commodities
$c \in CE (\subset C)$	exported commodities
$c \in CNE (\subset C)$	non-exported commodities
$f \in F$	factors
$h \in H (\subset I)$	households
$i \in I$	institutions (households, government, and rest of world)

Parameters

ad_a	production function efficiency parameter
aq_c	shift parameter for composite supply (Armington) function
at_c	shift parameter for output transformation (CET) function ²⁰
cpi	consumer price index
$cwts_c$	commodity weight in CPI
ica_{ca}	quantity of c as intermediate input per unit of activity a
mps_h	share of disposable household income to savings
pwe_c	export price (foreign currency)

¹⁹ Neither of the two commodities are both exported and imported. The phenomenon of two-way trade ("cross-hauling") is nevertheless commonly observed in the real world at the level of commodity aggregation used in applied models. It can be handled by the proposed approach without any modifications in the model structure.

²⁰ The acronym CET stands for constant elasticity of transformation.

pwm_c	import price (foreign currency)
qg_c	government commodity demand
\overline{qinv}_c	base-year investment demand
$shry_{hf}$	share for household h in the income of factor f
te_c	export tax rate
tm_c	import tariff rate
tq_c	sales tax rate
$tr_{i' i}$	transfer from institution i' to institution i
ty_h	rate of household income tax
$\alpha_{f a}$	value-added share for factor f in activity a
$\beta_{c h}$	share of consumption of household h for commodity c
δ_c^q	share parameter for composite supply (Armington) function
δ_c^t	share parameter for output transformation (CET) function
θ_{ac}	yield of commodity c per unit of activity a
ρ_c^q	exponent ($-1 < \rho_c^q < \infty$) for composite supply (Armington) function
ρ_c^t	exponent ($1 < \rho_c^t < \infty$) for output transformation (CET) function
σ_c^q	elasticity of substitution for composite supply (Armington) function
σ_c^t	elasticity of transformation for output transformation (CET) function

Variables

EG	government expenditure
EXR	foreign exchange rate (domestic currency per unit of foreign currency)
$FSAV$	foreign savings
$IADJ$	investment adjustment factor
PA_a	activity price
PD_c	domestic price of domestic output
PE_c	export price (domestic currency)
PM_c	import price (domestic currency)
PQ_c	composite commodity price
PVA_c	value-added price
PX_c	producer price
QA_a	activity level
QD_c	quantity of domestic output sold domestically
QE_c	quantity of exports
$QF_{f a}$	quantity demanded of factor f by activity a
QFS_f	supply of factor f
$QH_{c h}$	quantity of consumption for household h of commodity c
$QINT_{ca}$	quantity of intermediate use of commodity c by activity a
$QINV_c$	quantity of investment demand

QM_c	quantity of imports
QQ_c	quantity supplied to domestic commodity demanders (composite supply)
QX_c	quantity of domestic output
$WALRAS$	dummy variable (zero at equilibrium)
WF_f	average wage (rental rate) of factor f
$WFDIST_{fa}$	wage distortion factor for factor f in activity a
YF_{hf}	transfer of income to household h from factor f
YG	government revenue
YH_h	household income

Equations

Price block

The introduction of foreign trade with product differentiation drastically enriches the price system — out of the six equations in this block, four (Equations 1-4) are new.

Import price

$$PM_c = (1 + tm_c) \cdot EXR \cdot pwm_c \quad c \in CM \quad (1)$$

$$\begin{bmatrix} \text{import} \\ \text{price} \\ (\text{dom. cur.}) \end{bmatrix} = \begin{bmatrix} \text{tariff} \\ \text{adjust -} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (\text{dom. cur. per} \\ \text{unit of for. cur.}) \end{bmatrix} \cdot \begin{bmatrix} \text{import} \\ \text{price} \\ (\text{for. cur.}) \end{bmatrix}$$

Export price

$$PE_c = (1 - te_c) \cdot EXR \cdot pwe_c \quad c \in CE \quad (2)$$

$$\begin{bmatrix} \text{export} \\ \text{price} \\ (\text{dom. cur.}) \end{bmatrix} = \begin{bmatrix} \text{tax} \\ \text{adjust -} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (\text{dom. cur. per} \\ \text{unit of for. cur.}) \end{bmatrix} \cdot \begin{bmatrix} \text{export} \\ \text{price} \\ (\text{for. cur.}) \end{bmatrix}$$

The exogeneity of foreign-currency import and export prices indicates that we are modeling a country that is small relative to the relevant world markets (the “small-country” assumption). Note that Equations 1 and 2 only apply to imported and exported commodities, respectively.

Absorption

$$PQ_c \cdot QQ_c = \left[PD_c \cdot QD_c + (PM_c \cdot QM_c) \Big|_{c \in CM} \right] \cdot (1 + tq_c) \quad c \in C$$
$$[absorption] = \left(\left[\begin{array}{c} \text{domestic sales price} \\ \text{times} \\ \text{domestic sales quantity} \end{array} \right] + \left[\begin{array}{c} \text{import price} \\ \text{times} \\ \text{import quantity} \end{array} \right] \right) \cdot \left[\begin{array}{c} \text{sales tax} \\ \text{adjustment} \end{array} \right] \quad (3)$$

For each commodity, absorption — total domestic spending on the commodity at domestic demander prices — is expressed as the sum of spending on domestic output and imports, including an upward adjustment for the sales tax. The fact that this condition holds follows from the linear homogeneity of the composite supply (Armington) function (Equation 11; the condition is referred to as Euler's theorem). The import part only applies to imported commodities. The composite price, PQ_c , is paid by domestic demanders (households, the government, producers, and investors); hence it replaces P_c in all relevant equations. The composite price, implicitly defined by this equation, could easily be derived by dividing through by QQ_c . (See discussion of Equations 11 and 12 for further details.)

Domestic output value

$$PX_c \cdot QX_c = PD_c \cdot QD_c + (PE_c \cdot QE_c) \Big|_{c \in CE} \quad c \in C \quad (4)$$
$$\left[\begin{array}{c} \text{producer} \\ \text{price} \end{array} \right] = \left[\begin{array}{c} \text{domestic sales price} \\ \text{times} \\ \text{domestic sales quantity} \end{array} \right] + \left[\begin{array}{c} \text{export price} \\ \text{times} \\ \text{export quantity} \end{array} \right]$$

For each commodity, domestic output value at producer prices is stated as the sum of the value of domestic output sold domestically and the export value (in domestic-currency). This equation reflects the fact that the CET (constant-elasticity-of-transformation) function (Equation 14) is linearly homogeneous. The export part only applies to exported commodities. The producer price, PX_c , can be derived by dividing through by QX_c . Note that, in this model, the domestic output quantity is referred to as QX_c (as opposed to Q_c in earlier models). See discussion of Equations 14 and 15 for further details.

Activity price

$$PA_a = \sum_{c \in C} PX_c \cdot \theta_{ac} \quad a \in A \quad (5)$$

$$\begin{bmatrix} \text{activity} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{producer prices} \\ \text{times yields} \end{bmatrix}$$

Value-added price

$$PVA_a = PA_a - \sum_{c \in C} PQ_c \cdot ica_{ca} \quad a \in A \quad (6)$$

$$\begin{bmatrix} \text{value -} \\ \text{added} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{activity} \\ \text{price} \end{bmatrix} - \begin{bmatrix} \text{input cost} \\ \text{per activity} \\ \text{unit} \end{bmatrix}$$

Note that, in the last two equations, there is a change in notation for producer and composite supply prices.

Production and commodity block

In this block, Equations 7-10 are unchanged (except for a minor notation change in Equation 10). Equations 11-16 are new. They allocate domestic supply of composite commodities between imports and domestic output, and transform domestic output to exports and domestic sales. Simpler expressions apply to commodities that are not imported and/or not exported.

Activity production function

$$QA_a = ad_a \cdot \prod_{f \in F} QF_{fa}^{\alpha_{fa}} \quad a \in A \quad (7)$$

$$\begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix} = f \begin{bmatrix} \text{factor} \\ \text{inputs} \end{bmatrix}$$

Factor demand

$$WF_f \bullet WFDIST_{fa} = \frac{\alpha_{fa} \bullet PVA_a \bullet QA_a}{QF_{fa}} \quad f \in F \quad a \in A \quad (8)$$

$$\begin{bmatrix} \text{marginal cost} \\ \text{of factor } f \\ \text{in activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue} \\ \text{product of factor} \\ f \text{ in activity } a \end{bmatrix}$$

Intermediate demand

$$QINT_{ca} = ica_{ca} \bullet QA_a \quad c \in C, a \in A \quad (9)$$

$$\begin{bmatrix} \text{inter-} \\ \text{mediate} \\ \text{demand} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$$

Output function

$$QX_c = \sum_{a \in A} \theta_{ac} \bullet QA_a \quad c \in C \quad (10)$$

$$\begin{bmatrix} \text{domestic} \\ \text{output} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$$

Composite supply (Armington) function

$$QQ_c = aq_c \left[\delta_c^q \cdot QM_c^{-\rho_c^q} + (1 - \delta_c^q) \cdot QD_c^{-\rho_c^q} \right]^{-\frac{1}{\rho_c^q}} \quad c \in CM \quad (11)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = f \begin{bmatrix} \text{import quantity, domestic} \\ \text{use of domestic output} \end{bmatrix}$$

The composite commodities are used by all domestic demanders. Imperfect substitutability between imports and domestic output sold domestically is captured by a CES (constant elasticity of substitution) aggregation function in which the composite commodity that is supplied domestically is "produced" by domestic and imported commodities, entering this function as "inputs." Economically, this means that demander preferences over imports and domestic output are expressed as a CES function. This function, with a domain that is limited to elements in CM, is often called an Armington function after the originator of the idea of using a CES function for this purpose (Armington, 1969). The restriction on the value of ρ_c^q ($-1 < \rho_c^q < \infty$) assures that the corresponding isoquant is convex to the origin, in terms of production economics equivalent to a

diminishing technical rate of substitution.

Import-domestic demand ratio

$$\frac{QM_c}{QD_c} = \left(\frac{PD_c}{PM_c} \cdot \frac{\delta_c^q}{1 - \delta_c^q} \right)^{\frac{1}{1 + \rho_c^q}} \quad c \in CM \quad (12)$$

$$\begin{bmatrix} \text{import -} \\ \text{domestic} \\ \text{demand ratio} \end{bmatrix} = f \begin{bmatrix} \text{domestic -} \\ \text{import} \\ \text{price ratio} \end{bmatrix}$$

Equation 12 defines the optimal mix between imports and domestic output. Its domain is also limited to imported commodities. Together, Equations 3, 11 and 12 constitute the first-order conditions for cost-minimization given the two prices and subject to the Armington function and a fixed quantity of the composite commodity.

Composite supply for non-imported commodities

$$QQ_c = QD_c \quad c \in CNM \quad (13)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{domestic use of} \\ \text{domestic output} \end{bmatrix}$$

For commodities that are not imported, the Armington function is replaced by the above statement which imposes equality between “composite” supply and domestic output used domestically.

Output transformation (CET) function

$$QX_c = at_c \left[\delta_c^t \cdot QE_c^{\rho_c^t} + (1 - \delta_c^t) \cdot QD_c^{\rho_c^t} \right]^{\frac{1}{\rho_c^t}} \quad c \in CE \quad (14)$$

$$\begin{bmatrix} \text{domestic} \\ \text{output} \end{bmatrix} = f \begin{bmatrix} \text{export quantity, domestic} \\ \text{use of domestic output} \end{bmatrix}$$

Imperfect substitutability between imports and domestic output sold domestically is paralleled by imperfect transformability between domestic output for exports and domestic sales, captured by Equation 14. The CET (constant-elasticity-of-transformation) function, which applies to exported commodities, is identical to a CES function except for negative elasticities of

substitution. The isoquant corresponding to the output transformation function will be concave to the origin given the restriction imposed on the value of ρ^t ($1 < \rho_c^t < \infty$). In economic terms, the difference between the Armington and CET functions is that the arguments in the former are inputs, those in the latter are outputs.

Export-domestic supply ratio

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PD_c} \cdot \frac{1 - \delta_c^t}{\delta_c^t} \right)^{\frac{1}{\rho_c^t - 1}} \quad c \in CE \quad (15)$$

$$\begin{bmatrix} \text{export -} \\ \text{domestic} \\ \text{supply ratio} \end{bmatrix} = f \begin{bmatrix} \text{export -} \\ \text{domestic} \\ \text{price ratio} \end{bmatrix}$$

Equation 15 defines the optimal mix between exports and domestic sales. Equations 4, 14 and 15 constitute the first-order conditions for maximization of producer revenues given the two prices and subject to the CET function and a fixed quantity of domestic output.

One important difference between the equations for import demand (12) and export supply (15) is that the quantity demanded of the imported commodity (QM_c) is inversely related to the import price whereas the quantity supplied of the exported commodity (QE_c) is directly related to the export price.

Output transformation for non-exported commodities

$$QX_c = QD_c \quad c \in CNE \quad (16)$$

$$\begin{bmatrix} \text{domestic} \\ \text{output} \end{bmatrix} = \begin{bmatrix} \text{domestic sales of} \\ \text{domestic output} \end{bmatrix}$$

For commodities that are not exported, the CET function is replaced by a statement imposing equality between domestic output sold domestically and domestic output.

Institution Block

This block is not modified except for the appearance of items related to interactions with the rest of the world (trade and transfers) in the definitions of household revenue and both the

revenue and the expenditures of the government.

Factor income

$$YH_{hf} = shry_{hf} \cdot \sum_{a \in A} WF_f \cdot WFDIST_{fa} \cdot QF_{fa} \quad h \in H, f \in F \quad (17)$$

$$\begin{bmatrix} \text{income of} \\ \text{household } h \\ \text{from factor } f \end{bmatrix} = \begin{bmatrix} \text{share of} \\ \text{household } h \text{ in} \\ \text{income of factor } f \end{bmatrix} \cdot \begin{bmatrix} \text{income of} \\ \text{factor } f \end{bmatrix}$$

Household income

$$YH_h = \sum_{f \in F} YF_{hf} + tr_{h,gov} + EXR \cdot tr_{h,row} \quad h \in H \quad (18)$$

$$\begin{bmatrix} \text{household} \\ \text{income} \end{bmatrix} = \begin{bmatrix} \text{factor} \\ \text{incomes} \end{bmatrix} + \begin{bmatrix} \text{transfers from} \\ \text{government \&} \\ \text{rest of world} \end{bmatrix}$$

Household consumption demand

$$QH_{ch} = \frac{\beta_{ch} \cdot (1 - mps_h) \cdot (1 - ty_h) \cdot YH_h}{PQ_c} \quad c \in C, h \in H \quad (19)$$

$$\begin{bmatrix} \text{household} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = f \begin{bmatrix} \text{household income,} \\ \text{composite price} \end{bmatrix}$$

Investment demand

$$QINV_c = \overline{qinv_c} \cdot IADJ \quad c \in C \quad (20)$$

$$\begin{bmatrix} \text{investment} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{base - year investment} \\ \text{times} \\ \text{adjustment factor} \end{bmatrix}$$

Government revenue

$$YG = \sum_{h \in H} ty_h \cdot YH_h + EXR \cdot tr_{gov,row} + \sum_{c \in C} tq_c \cdot (PD_c \cdot QD_c + (PM_c \cdot QM_c)_{c \in CM}) + \sum_{c \in CM} tm_c \cdot EXR \cdot pwm_c \cdot QM_c + \sum_{c \in CE} te_c \cdot EXR \cdot pwe_c \cdot QE_c \quad (21)$$

$$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{direct} \\ \text{taxes} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from RoW} \end{bmatrix} + \begin{bmatrix} \text{sales tax} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{import tariff} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{export tax} \\ \text{revenue} \end{bmatrix}$$

Government expenditures

$$EG = \sum_{h \in H} tr_{h, gov} + \sum_{c \in C} PQ_c \cdot qg_c \quad (22)$$

$$\begin{bmatrix} \text{government} \\ \text{spending} \end{bmatrix} = \begin{bmatrix} \text{household} \\ \text{transfers} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix}$$

System constraint block

This block defines the constraints that are satisfied by the economy as a whole without being considered by its individual agents. The model's micro constraints apply to individual markets for factors and commodities. With a few exceptions discussed below (for labor, exports and imports), it is assumed that flexible prices clear the markets for all commodities and factors. The macro constraints apply to the government, the savings-investment balance, and the rest of the world. For the government, savings clear the balance, whereas the investment value adjusts to changes in the value of total savings. For the rest of the world, the alternatives of a flexible exchange rate or flexible foreign savings are permitted in the current formulation.

In this block, the rest-of-world constraint (Equation 25) is new while the commodity market and savings-investment balance (Equations 24 and 26) have been modified. The treatment of factor markets (Equation 23) is unchanged.

Factor markets

$$\sum_{a \in A} QF_{fa} = QFS_f \quad f \in F \quad (23)$$

$$\begin{bmatrix} \text{demand for} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{supply of} \\ \text{factor } f \end{bmatrix}$$

For the two factors, the closure rules are the same as for Exercise 4: unemployment with fixed, activity-specific real wages for labor and fixed capital use for each activity. This is achieved by fixing the following variables at base values: $WFDIST_{lab, a}$, WF_{lab} , $QF_{cap, a}$ and WF_{cap} .

Composite commodity markets

$$QQ_c = \sum_{a \in A} QINT_{ca} + \sum_{h \in H} QH_{ch} + qg_c + QINV_c \quad c \in C \quad (24)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{composite demand : sum of intermediate,} \\ \text{household, government and investment demand} \end{bmatrix}$$

In the absence of foreign trade, the commodity market equilibrium condition in Exercises 1-4 equated output and domestic demand. This new equilibrium condition imposes equality in the composite commodity market with the demand side represented by all types of domestic commodity use while the supply comes from the Armington function (or its substitute for non-imported commodities) that aggregates imports and domestic output sold domestically. The variable PQ_c clears this market.

In addition to the composite commodity, the model includes quantity (and associated price) variables for the following commodities and activities: QM, QE, QX, QD, QA. These variables represent both the quantities supplied and demanded (i.e., the equilibrium quantity has been substituted for the quantities supplied and demanded throughout the model). For exports and imports, the quantities demanded and supplied clear the markets (infinitely elastic world market demands and supplies at fixed foreign-currency prices). For the remaining three quantities, the associated price variables (PX, PD, and PA) serve the market-clearing role. (Exercise: Rewrite the model with separate supply and demand variables replacing QM, QE, QX, QD, QA, and a full set of equilibrium conditions for the corresponding markets.)

Current account balance for RoW (in foreign currency)

$$\sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in I} tri_{i,row} + FSAV = \sum_{c \in CM} pwm_c \cdot QM_c \tag{25}$$

$$\begin{bmatrix} \textit{export} \\ \textit{revenue} \end{bmatrix} + \begin{bmatrix} \textit{transfers} \\ \textit{from RoW} \\ \textit{to households} \\ \textit{\& government} \end{bmatrix} + \begin{bmatrix} \textit{foreign} \\ \textit{savings} \end{bmatrix} = \begin{bmatrix} \textit{import} \\ \textit{spending} \end{bmatrix}$$

The current-account equation (which is expressed in foreign currency) imposes equality between the country's earning and spending of foreign exchange. Foreign savings is equal to the current-account deficit. Careful counting of equations and variables in the current model would indicate that the number of variables exceeds the number of equations by one. This is related to the fact that the model includes two variables that may serve the role of clearing the current-account balance — the foreign exchange rate (EXR) and foreign savings (FSAV). The experiment for this Exercise (see below), assumes that FSAV is fixed.

Savings-investment balance

$$\sum_{h \in H} mps_h \cdot (1 - ty_h) \cdot YH_h + (YG - EG) + EXR \cdot FSAV = \sum_{c \in C} PQ_c \cdot QINV_c + WALRAS \quad (26)$$
$$\begin{bmatrix} \text{household} \\ \text{savings} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix} = \begin{bmatrix} \text{investment} \\ \text{spending} \end{bmatrix} + \begin{bmatrix} \text{WALRAS} \\ \text{dummy variable} \end{bmatrix}$$

Foreign savings, converted into domestic currency, appear as a new item in Equation 26. As long as either the exchange rate or foreign savings is fixed, its presence does not influence the savings-investment closure of the model according to which the savings value determines the investment value.

Price normalization

$$\sum_{c \in C} cwtsc_c \cdot PQ_c = cpi \quad (27)$$
$$[\text{weighted sum of prices}] = [\text{consumer price index}]$$

TASKS

Implement the model presented in the mathematical statement in GAMS, i.e., calibrate it to the base, solve it to confirm that it calibrates, and implement a simple experiment where foreign savings is doubled.

As your starting point, use the file CGE4.GMS (the suggested answer to Exercise 4) and relevant parts of CGEHLP.TXT. The latter includes the new SAM, as well as declarations and definitions for new parameters, variables, and equations. However, the numerous changes in definitions for old equations, parameters and variables are not included.

HINTS AND SUGGESTIONS

It is suggested that you go through the following steps:

1. Carefully review the file CGE4.GMS, the mathematical statement for the Exercise 5 model, and the relevant part of CGEHLP.TXT
2. Make a copy of the file CGE4.GMS named CGE5.GMS
3. Carefully work through the file CGE5.GMS starting from the very beginning, introducing modifications when implied by the mathematical statement and the new

SAM, copying segments from CGEHLP.TXT. In particular, it should be helpful to copy and paste the relatively complex definitions of parameters related to Armington and CET functions.

4. When encountering problems, draw on suggestions in earlier Exercises regarding how to debug the model. Rely on the GAMS User's Guide as a reference.
5. Regarding initial values for price variables, it may be useful to note the following: In general, we try to initialize as many prices as possible at unity. In the current model, this is the case for all factor prices and for the commodity prices PE_c , PM_c , PD_c , and PX_c . However, the possible presence of taxes and subsidies may require divergence from this initialization rule for three other commodity prices: $tq_c > 0 \Rightarrow PQ_c > 1$; $tm_c > 0 \Rightarrow pwm_c < 1$; and $te_c > 0 \Rightarrow pwe_c > 1$. (However, according to the current SAM, $te_c = 0$ for all commodities.)

The suggested GAMS model has 49 variables and equations.

GOOD LUCK!

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