A Recursive Dynamic CGE Model and Microsimulation Poverty Module for South Africa

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1. Introduction

Computable general equilibrium (CGE) models are widely used for policy-analysis in many countries. In the past a number of CGE models have been developed for South Africa, and used to assess a broad range of policy issues.[1] However, the perceived complexity of this analytical approach, and the concentration of capacity within a small number of academic or related institutions, have generally led policy-makers, analysts and other researchers to avoid directly using CGE models in their analysis or decision-making. Since CGE modelling provides both an economy-wide assessment of policies and a framework in which the workings of policies can be more easily understood, it is the objective of this paper to present a core South African model that reduces the initial cost of undertaking CGE analysis. The core model can then be adapted according to the interests of individual researchers or policy-makers. Furthermore, since the strength of the model is dependent on its ability to reflect the specific structure and workings of the South African economy, it is hoped that the core model will be developed further as more supporting evidence and research becomes available.

The model presented in this paper has at its core the static model used by the International Food Policy Research Institute (IFPRI) as described in Lofgren *et al*. (2002). The model is recursive dynamic and is therefore an extension of the IFPRI model and the earlier static South African model presented in Thurlow and van Seventer (2002). This revision of the documentation to support the model differ from the earlier recursive dynamic model for South Africa (see Thurlow, 2004) in that includes a microsimulation poverty module and now makes better use of GAMS GDX facilitaties.

In the South African model, the structure and interactions of the economy within and across time periods is specified in a set of mathematical equations. Section 2 describes the specification and limitations of the South African model without the aid of mathematics. Since the underlying static South African model is essentially that of the IFPRI standard model, Section 3 first presents the differences in the mathematical equations between these two models, before describing the mathematics of the model's dynamic specification.

Finally, Section 4 concludes the paper by describing existing applications of the models and identifying areas where further research is needed to address the limitations of the model.

[1] See Thurlow and van Seventer (2002) for a brief review of past economy-wide modeling in South Africa.

2. Model Description

The dynamic South African model described below has developed from the neoclassical-structuralist modelling tradition originally presented in Dervis *et al* (1982), and has at its core the static CGE model described in Lofgren *et al* (2002) and Thurlow and van Seventer (2002). The model is formulated as a set of simultaneous linear and non-linear equations, which define the behaviour of economic agents, as well as the economic environment in which these agents operate. This environment is described by market equilibrium conditions, macroeconomic balances, and dynamic updating equations.

The model belongs to the recursive dynamic strand of the dynamic CGE literature, which implies that the behaviour of its agents is based on adaptive expectations, rather than on the forward-looking expectations that underlie alternative inter-temporal optimisation models. Since a recursive model is solved one period at a time, it is possible to separate the *within-period* component from the *between-period* component, where the latter governs the dynamics of the model. Although a detailed mathematical description can be found in Appendix A and in Lofgren *et al* (2002), this section presents a more discursive overview of the model's structure.[2]

2.1 Within-period Specification

The within-period component describes a one-period static CGE model. The following description of this model is divided into the derivation of production and prices, and the generation of institutional incomes and demand. Equilibrium is maintained through a series of system constraints which are discussed last.

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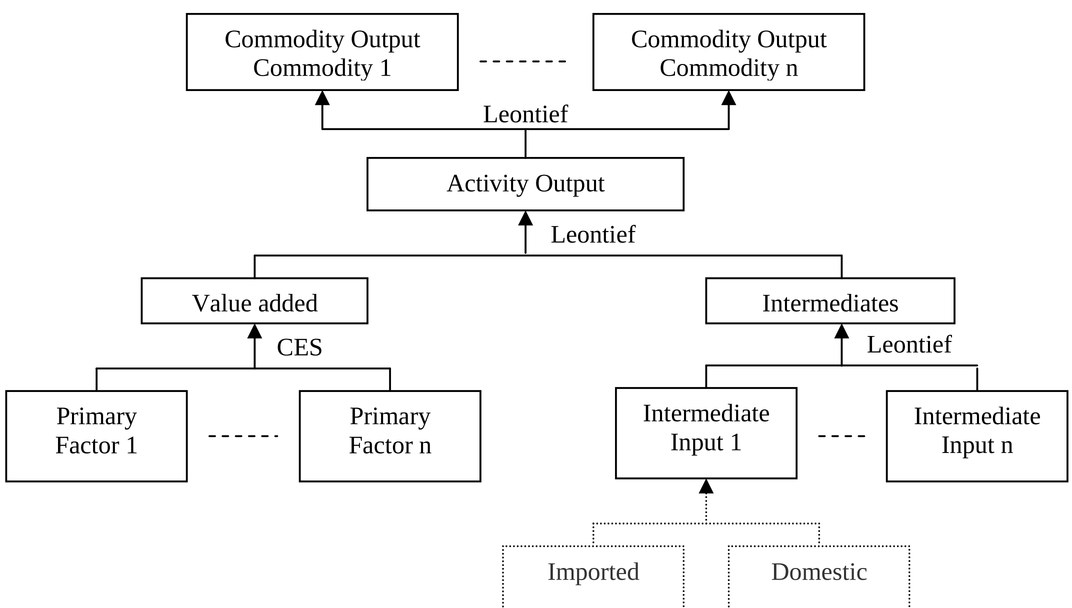
[2] The model and underlying data is available from Trade and Industrial Policy Strategies (www.tips.org.za) or from the author (j.thurlow@cgiar.org).

Production and Prices

The model identifies 43 productive sectors or activities that combine primary factors with intermediate commodities to determine a level of output. The four factors of production identified in the model include capital, unskilled and semi-skilled, skilled, and highly-skilled labour.[3] The technology underlying production is depicted for a single producer in Figure 2.1. Producers in the model make decisions in order to maximize profits subject to constant returns to scale, with the choice between factors being governed by a constant elasticity of substitution (CES) function. This specification allows producers to respond to changes in relative factor returns by smoothly substituting between available factors so as to derive a final value-added composite. Profit-maximization implies that the factors receive income where marginal revenue equals marginal cost based on endogenous relative prices. Once determined, these factors are combined with fixed-share intermediates using a Leontief specification. The use of fixed-shares reflects the belief that the required combination of intermediates per unit of output, and the ratio of intermediates to value-added, is determined by technology rather than by the decision-making of producers. The final price of an activity's output is derived from the price of value-added and intermediates, together with any producer taxes or subsidies that may be imposed by the government per unit of output.

Figure 2.1: Production Technology[1]



1 'CES' is a constant elasticity of substitution aggregation function. 'Leontief' is fixed shares.

[3] A detailed account of the different factor categories is provided in Section 3.

In addition to its multi-sector specification, the model also distinguishes between activities and the commodities that these activities produce. This distinction allows individual activities to produce more than a single commodity and conversely, for a single commodity to be produced by more than one activity.[4] Fixed-shares govern the disaggregation of activity output into commodities since it is assumed that technology largely determines the production of secondary products. These commodities are supplied to the market.

Figure 2.2 traces the flow of a single commodity from being supplied to the market to its final demand. The previous figure showed how a single producer could supply more than one of the 43 commodities identified by the model. In the figure below, the supply of a particular commodity from each producer is combined to derive aggregate commodity output. This aggregation is governed by a CES function which allows demanders to substitute between the different producers supplying a particular commodity, in order to maximise consumption subject to relative supply prices.

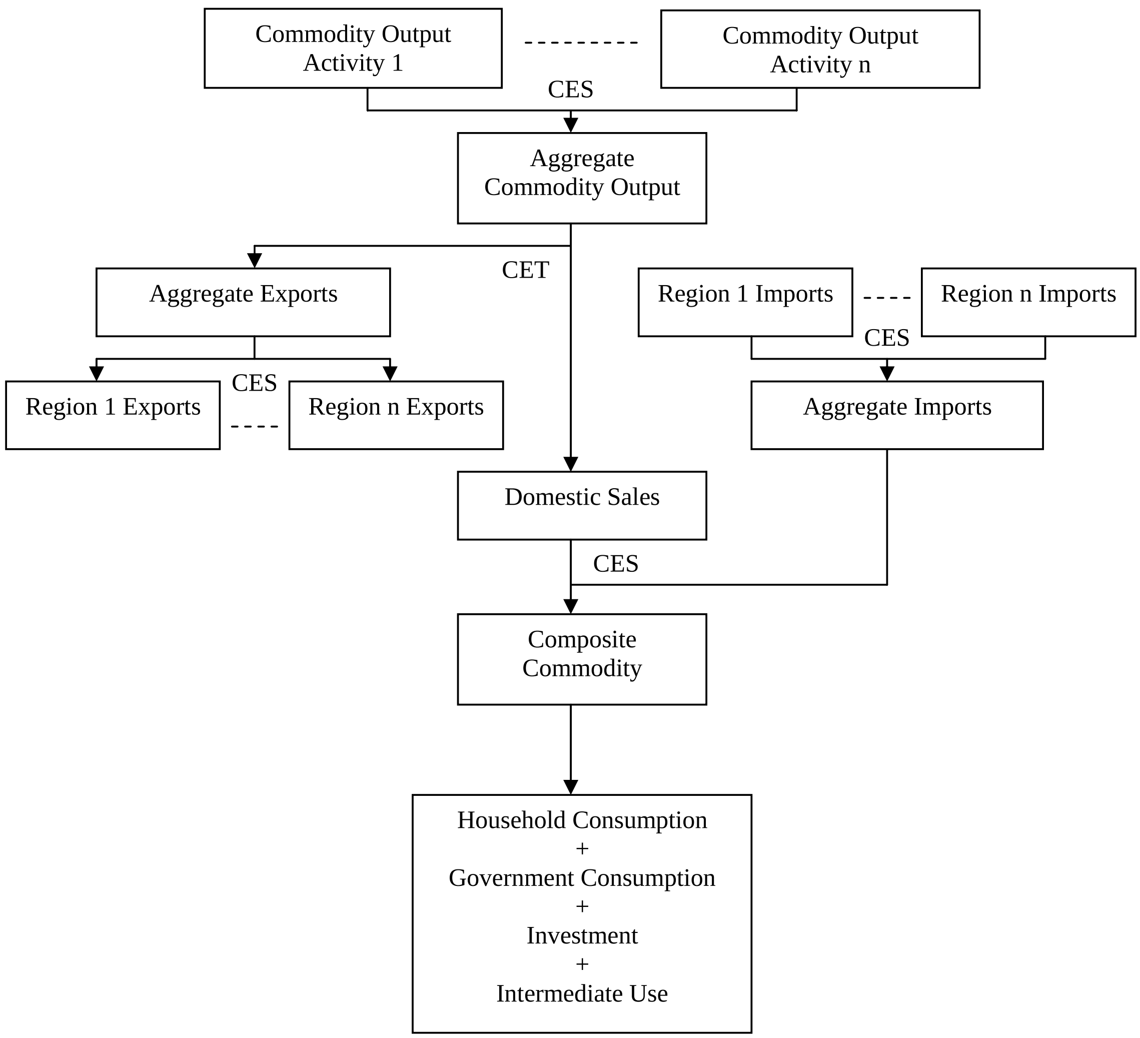
Substitution possibilities exist between production for the domestic and the foreign markets. This decision of producers is governed by a constant elasticity of transformation (CET) function, which distinguishes between exported and domestic goods, and by doing so, captures any time or quality differences between the two products. Profit maximization drives producers to sell in those markets where they can achieve the highest returns. These returns are based on domestic and export prices (where the latter is determined by the world price times the exchange rate adjusted for any taxes or subsidies). Under the small-country assumption, South Africa is assumed to face a perfectly elastic world demand at a fixed world price. The final ratio of exports to domestic goods is determined by the endogenous interaction of relative prices for these two commodity types. Commodities that are exported are further disaggregated according to their region of destination under a CES specification. Allowing substitution between regions is preferable to the use of fixed shares, since changes in relative prices across regions should lead to a shift in the geographic composition of exports.

Domestically produced commodities that are not exported are supplied to the domestic market. Substitution possibilities exist between imported and domestic goods under a CES Armington specification (Armington, 1969). Such substitution can take place both in final and intermediates

[4] For example, although the agricultural sector's primary output is agricultural products, this sector might also produce some processed food products. Therefore this single sector or activity can produce more than one product or commodity. Conversely, since food is also produced by the processed food sector, the combination of agricultural and processed food production suggests that some commodities can also be produced by more than one activity.

usage. The Armington elasticities vary across sectors, with lower elasticities reflecting greater differences between domestic and imported goods.[5] Again under the small country assumption, South Africa is assumed to face infinitely elastic world supply at fixed world prices. The final ratio of imports to domestic goods is determined by the cost minimizing decision-making of domestic demanders based on the relative prices of imports and domestic goods (both of which include relevant taxes). Imports are further disaggregated according to their region of origin using a CES function. This specification allows for regionally specific tariffs, and for substitution between regions following changes in relative import prices.

Figure 2.2: Commodity Flows[1]



1 'CES' is a constant elasticity of substitution aggregation function. 'CET' is constant elasticity of transformation function.

[5] The use of an Armington specification is justified by the likely heterogeneity of commodities within broad commodity categories, and by the observed two-way trade between South Africa and its trading partners. See Section 3 and Appendix C for the values of the Armington elasticities used in the model.

Transaction costs are incurred on exports, imports and domestic sales. These costs are treated as a fixed share per unit of commodity, and generate demand for trade and transportation services. The final composite good, containing a combination of imported and domestic goods, is supplied to both final and intermediate demand. Intermediate demand, as described above, is determined by technology and by the composition of sectoral production. Final demand is dependent on institutional incomes and the composition of aggregate demand.

Institutional Incomes and Domestic Demand

The model distinguishes between various institutions within the South African economy, including enterprises, the government, and 14 types of households. The household categories are disaggregated across income deciles with the exception of the top decile, which has five income divisions. Figure 2.3 summarises the interaction between institutions in the model.

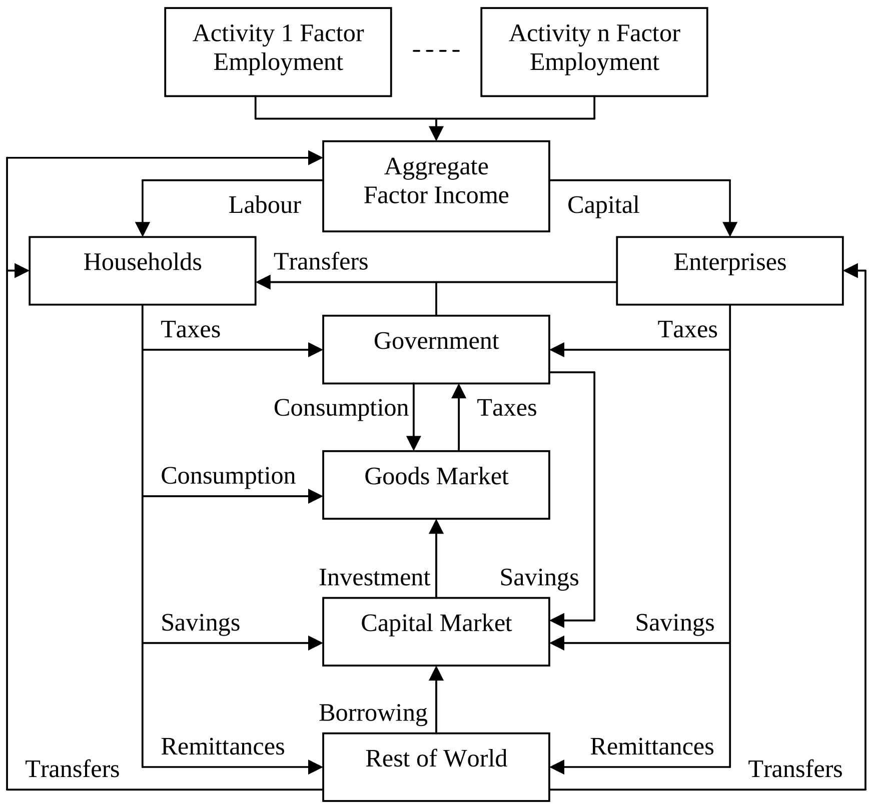
The primary source of income for households and enterprises are factor returns generated during production. The supply of capital is fixed within a given time-period and is immobile across sectors, thus implying that capital earns sector-specific returns. Unskilled and semi-skilled, and skilled labour supply is assumed to be perfectly elastic at a given real wage. Highly-skilled labour face upward-sloping labour supply curves, with wage elasticities determining adjustments to supply following changes in real wages.[6] Each activity pays an activity-specific wage that is the product of the economy-wide wage and a fixed activity-specific wage distortion term. This specification, in which factor returns are sector-specific, is preferable to the use of simple average wages, since average factor returns in South Africa are observed to vary both across occupations and sectors. Final factor incomes also include remittances received from and paid to the rest of the world.

Households and enterprises earn factor incomes in proportion to the implied share that they control of each factor stock. Enterprises or firms are the sole recipient of capital income, which they transfer to households after having paid corporate taxes (based on fixed tax rates), saved (based on fixed savings rates), and remitted profits to the rest of the world. Households within each income category are assumed to have identical preferences, and are therefore modelled as 'representative' consumers. In addition to factor returns, which represent the bulk of household incomes, households also receive transfers from the government, other domestic institutions, and the rest of the world. Household disposable income is net of personal income tax (based on fixed rates), savings (based

[6] The motivation for adopting these labour market closures for each of the three labour categories is presented in Section 3.

on fixed marginal propensities), and remittances to the rest of the world. Consumer preferences are represented by a linear expenditure system (LES) of demand, which is derived from the maximization of a Stone-Geary utility function subject to a household budget constraint. Given prices and incomes, these demand functions define households' real consumption of each commodity. The LES specification allows for the identification of supernumerary household income that ensures a minimum level of consumption.

Figure 2.3: Institutional Incomes and Domestic Demand



The government earns most of its income from direct and indirect taxes, and then spends it on consumption and transfers to households. Both of these payments are fixed in real terms. The difference between revenues and expenditures is the budget deficit, which is primarily financed through borrowing (or dis-saving) from the domestic capital market. Although not shown in Figure 2.3, the government also makes payments to the rest of the world. In the current model the government's role as a consumer is treated separately from the production of government services. The latter is specified as an activity producing services for which the government institution is the primary consumer.

Savings by households and enterprises are collected into a savings pool from which investment is financed. This supply of loanable funds is diminished by government borrowing (or dis-saving) and augmented by capital inflows from the rest of the world. There is no explicit modelling of the investment decision or the financial sector within a particular time-period, with savings equalling investment as per the ex post accounting identity. This implicitly assumes that the necessary adjustment in the interest rate takes place to ensure that savings equals investment in equilibrium. The disaggregation of investment into demand for final commodities is done using fixed shares, with changes in aggregate investment leading to proportional increases in the demand for individual commodities. Therefore there is no real compositional shift in investment following changes in relative commodity prices.

Production is linked to demand through the generation of factor incomes and the payment of these incomes to domestic institutions. Balance between demand and supply for both commodities and factors are necessary in order for the model to reach equilibrium. This balance is imposed on the model through a series of system constraints.

System Constraints and Macroeconomic Closures

Equilibrium in the goods market requires that demand for commodities equal supply. Aggregate demand for each commodity comprises household and government consumption spending, investment spending, and export and transaction services demand. Supply includes both domestic production and imported commodities. Equilibrium is attained through the endogenous interaction of domestic and foreign prices, and the effect that shifts in relative prices have on sectoral production and employment, and hence institutional incomes and demand.

The equilibrating of factor demand and supply is dependent on how the relationship between factor supply and wages is defined. As discussed above, capital is fully employed and sector-specific, implying that sector-specific wages adjust to ensure that demand for capital equals total supply. Unemployment amongst unskilled and semi-skilled, and skilled labour is assumed to be sufficiently large such that wages are fixed in real terms and supply passively adjusts to match demand. Highly-skilled labour is neither fully employed nor significantly unemployed to justify either a fixed supply or a fixed wage. Rather the supply of this factor is responsive to changes in real wages, which adjust to ensure that demand and supply are equal in equilibrium.

The model includes three broad macroeconomic accounts: the current account, the government balance, and the savings and investment account. In order to bring about equilibrium in the various macro accounts it is necessary to specify a set of 'macroclosure' rules, which provide a mechanism through which adjustment is assumed to take place.

For the current account it is assumed that a flexible exchange rate adjusts in order to maintain a fixed level of foreign borrowing (or negative savings). In other words, the external balance is held fixed in foreign currency. This closure is appropriate given South Africa's commitment to a flexible exchange rate system, and the belief that foreign borrowing is not inexhaustible. However given movements in South Africa's current account balance, it might be necessary to exogenously adjust foreign savings based on observed trends and let the exchange rate adjust accordingly.

In the government account the level of direct and indirect tax rates, as well as real government consumption, are held constant. As such the balance on the government budget is assumed to adjust to ensure that public expenditures equal receipts. This closure is chosen since it is assumed that changes in direct and indirect tax rates are politically motivated and thus are adopted in isolation of changes in other policies or the economic environment.

Although the government and current account closures can be selected based on current government policies, the choice of a savings-investment closure is less obvious. According to Nell (2003), the relationship between saving and investment remains one of the most debated and controversial issues in macroeconomics. On the one hand, neoclassical and recent endogenous growth theory maintains that it is prior savings that is most important when determining an economy's level of investment and output. This view suggests that savings is exogenous, and that investment adjusts passively to maintain the savings-investment balance. By contrast, a more Keynesian view reverses the causality found in neoclassical theory by arguing that investment is exogenous and that it is savings that adjusts. Finally, there might exist, as in the case of some developed countries, a two-way causality between savings and investment. In such cases both the level of savings and investment are endogenously determined and may both adjust in response to policy-changes.

The choice of which direction of causality is appropriate for South Africa might have implications for the outcomes of policies. For example, under the more neoclassical approach and in the case trade liberalization, a reduction in tariff revenue will decrease the level of government savings and thereby crowd-out private investment. Under the exogenous investment paradigm, maintaining the level of investment would require that savings would have to increase through increases in domestic savings rates. In such a case, the level of disposable income is reduced with ‘crowding-out’ effects on private consumption.

Recent work on this issue concluded that the long-run savings-investment relationship in South Africa has been one characterized by exogenous savings with no feedback from investment (Nell, 2003). Therefore the model adopts a savings-driven closure, in which the savings rates of domestic institutions are fixed, and investment passively adjusts to ensure that savings equals investment spending in equilibrium. However, the inclusion of dynamics into the model allows past investment to influence economic growth in the economy, and thereby the level of savings available for investment in the current period. The dynamics of the model are discussed below.

Finally, the consumer price index is chosen as the numéraire such that all prices in the model are relative to the weighted unit price of households’ initial consumption bundle. The model is also homogenous of degree zero in prices, implying that a doubling of all prices does not alter the real allocation of resources.

2.2 Between-period Specification

While the static model described above is detailed in its representation of the South African economy within a particular time-period, its inability to account for second-period considerations limits its assessment of the full effect of policy and non-policy changes. For example, the model is unable to account for the second-period effect that changes in current investment have on the subsequent availability of capital. In attempting to overcome these limitations, the static model is extended to a recursive dynamic model in which selected parameters are updated based on the modelling of inter-temporal behaviour and results from previous periods. Current economic conditions, such as the availability of capital, are thus endogenously dependent on past outcomes, but remain unaffected by forward-looking expectations. The dynamic model is also exogenously updated to reflect demographic and technological changes that are based on observed or separately calculated projected trends.

The process of capital accumulation is modelled endogenously, with previous-period investment generating new capital stock for the subsequent period. Although the allocation of new capital across sectors is influenced by each sector’s initial share of aggregate capital income, the final sectoral allocation of capital in the current period is dependent on the capital depreciation rate and on sectoral profit-rate differentials from the previous period. Sectors with above-average capital returns receive a larger share of investible funds than their share in capital income. The converse is true for sectors where capital returns are below-average.[7]

Population growth is exogenously imposed on the model based on separately calculated growth projections. It is assumed that a growing population generates a higher level of consumption demand and therefore raises the supernumerary income level of household consumption. There is assumed to be no change in the marginal rate of consumption for commodities, implying that new consumers have the same preferences as existing consumers.

Highly-skilled labour supply adjusts endogenously across periods in response to continuing changes in real wages. Between periods there may be an exogenous adjustment to the supply of this labour category as is typical in most recursive dynamic models. This treatment of the model's labour supply dynamics assumes that for the highly-skilled labour category there is neither a binding supply-constraint nor involuntary unemployment. Rather labour supply is seen as being driven by changes in real wages, thus suggesting the existence of an effective reservation wage.

Unskilled and semi-skilled, and skilled labour supply within a particular time period is infinitely elastic at a fixed real wage. As such it is the real wage, rather than labour supply, that adjusts between periods. In the dynamic model it is assumed that real wage changes for unskilled and skilled workers are relative to previous period changes in the real wage of highly skilled workers. This specification allows for the endogenous determination of wages for lower skilled workers, as well as the exogenous determination of skilled-unskilled wage convergence rates.[8]

Factor-specific productivity growth is imposed exogenously on the model based on observed trends for labour and capital. Growth in real government consumption and transfer spending is also exogenously determined between periods, since within-period government spending is fixed in real terms. Furthermore, projected changes in the current account balance are exogenously accounted for. Finally, mining production is assumed to be predominantly driven by a combination of changes in world demand and prices, and other factors external to the model. One such external factor might be the gradual exhaustion of non-renewable natural resources. Accordingly, the value-added growth

[7] See Dervis *et al* (1982) for a more detailed discussion of this and other approaches to modelling capital accumulation in CGE models.

[8] Exogenously imposed wage convergence (or divergence) suggests that there are there are factors outside of the model that are important in determining wages for unskilled and semi-skilled, and skilled workers. These factors might include the effective bargaining of trade unions or changes in South Africa's labour laws. As will be discussed in Section 3, observed wage convergence between highly-skilled and less-skilled workers justifies the current specification.

of these sectors and the world price of exports are updated exogenously between periods based on observed long-term trends.[9]

The South African dynamic model is solved as a series of equilibriums, each one representing a single year. By imposing the above policy-independent dynamic adjustments, the model produces a projected or counterfactual growth path. Policy changes can then be expressed in terms of changes in relevant exogenous parameters and the model is re-solved for a new series of equilibriums. Differences between the policy-influenced growth path and that of the counterfactual can then be interpreted as the economy-wide impact of the simulated policy.

2.3 Limitations of the Model

Applied general equilibrium modelling is an important tool for policy-analysis given that it is able to isolate the effects of individual policies, while explicitly specifying the causal mechanisms through which policies influence the economy. The CGE approach has advantages over data-based econometric analysis, which not only requires considerable and reliable time-series data, but also faces difficulties in isolating the effects of individual policies from other changes in policies and external factors. Furthermore, the sectoral and institutional detail of the CGE model allows for a more detailed analysis of policies than is typically possible with macro-econometric models. Finally, CGE models have an advantage over partial equilibrium analysis in that they offer an economy-wide assessment of policies, including the concurrent effects of policy-changes on production, employment, and poverty and inequality.

However, while economy-wide models have certain advantages over other methods of analysis, these models are more closely tied to theory, which often incorporates or necessitates an abstraction from the real workings of an economy. Therefore it is important to identify and account for the limitations of the model, especially in terms of its ability to reflect the country-specific characteristics of the economy being studied.

[9] Exogenously imposing a factor growth rate on a sector requires adjusting the capital accumulation process. For example, reducing mining output when capital is sectorally fixed leads to increases in mining capital's profit-rate. Since new capital allocation is driven by sectoral profit-rate differentials, a mining high profit-rate will therefore attract new investment. The mining sector therefore is excluded from the capital allocation decision after adjusting the stock of new capital to account for depreciation and fixed capital changes taking place within the mining sector.

Static and Dynamic Equilibrium

Perhaps the main criticism of the static model is that its core formulation is closely tied to the Walrasian ideal of equilibrium (Dervis *et al*, 1982). In a pure neoclassical setting, producers and consumers react passively to prices in order to determine their demand and supply schedules. Markets are therefore assumed to clear through the interaction of relative prices, such that equilibrium is achieved in both goods and factor markets. However, it might be argued that that certain institutional and structural rigidities within the South African economy result in cases of persistent disequilibrium or deviations from neoclassical theory.

The South African model does attempt to incorporate some the perceived rigidities in the economy's factor markets. For example, capital is assumed to be immobile across sectors, and unskilled and semi-skilled, and skilled labour supply is unemployed at a fixed real wage. Furthermore, factor returns are assumed to vary across sectors based on observed and persistent sectoral deviations from economy-wide averages. These rigidities allow for a 'constrained' general equilibrium that, while remaining close to the Walrasian model, accounts for some of the observed structural characteristics of the economy. However, Dervis *et al*. (1982) note that the adoption of a more Walrasian approach leads to problems in both factor *and* product markets. In the case of the latter, the South African model retains a neoclassical specification, and ignores such considerations as the existence of imperfect competition and monopoly-pricing.

The model assumes there is no interaction between monetary and real economies. The use of a numéraire and the lack of an explicitly modelled monetary sector imply that the model is essentially one of a barter economy in which money is neutral. Taylor (1983), in outlining the structuralist approach, discounts money-neutrality by arguing that nominal changes can influence the real economy, particularly within the short-run and in respect to the demand for money balances. Dervis *et al*. (1982) suggest however that, while separability is not always possible to preserve, the overall strength of the CGE approach lies in its ability to address questions of medium to long-term resource allocation.

The specification of capital accumulation and allocation within the dynamic model also represents a deviation from the perfect neoclassical inter-temporal equilibrium. Within the neoclassical framework, market and production prices of capital are identical, within-period sectoral profit-rates are equalised, and the economy moves along an inter-temporally efficient path characterised by perfect foresight (Dervis *et al*., 1982). However, in the adaptive dynamic South African model, capital is immobile across sectors and the allocation of new capital is partly determined by the distribution of previous-period capital incomes. Together these rigidities prevent both a within- and between-period equalisation of sectoral profit-rates. By not determining the inter-temporally efficient allocation of capital the model greatly simplifies the investment allocation decision, and avoids having to explicitly model expectations. This specification can be justified on the grounds that agents within the South African economy are unlikely to possess perfect foresight, and as such, the inter-temporal efficient growth path is unlikely to be achieved.

Given the institutional and structural rigidities of the South African economy, the use of a more neoclassical market-clearing mechanism suggests that caution be exercised in interpreting the model's results. Most importantly, the model is not able to provide short-term predictions, but rather highlights the direction and relative magnitude of adjustments to the economy following changes in policies, technology, and other external factors.

Production and Factor Demand

Production within the South African model is governed by neoclassical production functions, which may not reflect the specific workings of individual sectors. The model assumes constant returns to scale, and models 'representative' sectors such that all producers within each sector are assumed to share the same behaviour. Capital and labour are treated as equally substitutable for one another, thus implying, for example, that unskilled labour is as substitutable for capital as is highly-skilled labour. Finally, all producers are assumed to be on their factor demand curve. This last assumption rules out the possibility of excess capacity and the hoarding of labour during economic downturns. Although it is possible to adopt more flexible specifications of production, such as translog or nested-CES functions, these formulations require considerably more parameter estimates than are currently available for South Africa. Furthermore, the relatively high sectoral and factor aggregation of the model, and its medium to long-term focus, are likely to lessen the severity of the above limitations. For example, higher sectoral aggregation reduces the likelihood of monopoly-power within an individual sector.

Final Demand

Final household, government, and investment demand for each commodity is assumed to be a fixed share of aggregate institutional spending. Therefore expenditure shares for each commodity are fixed and do not adjust in response to changes in relative prices. While this is unlikely to reflect actual institutional behaviour, the use of fixed shares is preferable to the use of a more flexible functional form since short and medium-term substitution possibilities are likely to be limited. Furthermore, there is no existing information on South Africa that could inform the calibration of such behaviour.

This specification also does not allow household consumption patterns to adjust following changes in household incomes. The assumption that there is no income effect on final demand, or that the income elasticity of demand is unity, is unlikely to reflect reality. However, there is little reason to suspect that consumption patterns will adjust significantly as long as the time-period over which the model is used remains relatively short and income changes are small.

Foreign Trade

The model assumes that imports, exports, and domestic goods are imperfect substitutes. This assumption is more realistic than a 'perfect substitutes' specification, since the high sectoral aggregation of the model increases the likelihood of within-sector cross-hauling. However, in the case of imports, the allowance for differentiated products leads to the construction of a composite good containing both imported and domestic commodities. This marketed composite good is then supplied to all components of demand, thus assuming that all consumers of an individual commodity have the same import-intensity of consumption. For example, the import-share of the food composite is the same for low-income and high-income households. This is likely to overstate the import-intensity of low-income household food consumption, and understate high-income households' import-intensity.

By measuring trade policy using fixed tariff rates, the model does not explicitly account for the existence of quantitative restrictions or differential tariff rates that are determined by trade volumes. While the use of quantitative restrictions in South Africa had been greatly reduced prior to the beginning of the 1990s, South Africa's use of formula duties persisted into the 1990s, mainly within the agricultural and textiles sectors (Cassim *et al*, 2002). For these sectors the model assumes that tariff rates are fixed simple ad valorum rates that are unaffected by changes in import-quantities. Assuming that some tariff rates do increase as import volumes increase, the current specification is likely to understate tariff rates following increases in imports, and understate rates following declines in imports. However, Cassim *et al.* (2002) find that, even in the case of agriculture, collections rates are a good proxy for statutory rates, thereby lessening the likely severity of this limitation.

3. Model Specification

The South African model is an extension of the standard static model used by the International Food Policy Research Institute (IFPRI) (Lofgren *et al*, 2002). A number of equations have been added to the IFPRI model that allows (i) the regional disaggregation of international trade; (ii) an upward-sloping factor supply curve; and (iii) factor-specific productivity adjustments. The addition of these features requires that some of the existing equations in the IFPRI model be adjusted and new equations added. These changes to the static model are described in the first part of this section. In extending the static model to a recursive dynamic model a number of equations and updating procedures are included in the South African model. These are described in the second part. A complete listing of the model's variables, parameters and equations is provided in the appendix.

3.1 Additions to the Static Model

Regional Disaggregation of International Trade

Although it is not necessary to include regionally specific trade data in the South African model, the model's specification does allow for this additional information to be included during the calibration process. In the IFPRI model, imported and exported commodities were assigned to the sets *CM* and *CE* respectively. This assignment is retained in the South African model only for those commodities that are imported or exported but whose trade is *not* regional disaggregated. Imported and exported commodities that are regionally disaggregated are now assigned to the sets *CMR* and *CER*. These sets are two dimensional across commodities *and* regions, where the new set *R* contains a list of the trading regions included in the model. Although the set *R* contains regions for both imports and exports, it is not necessary for imports and exports to be disaggregated across the same regions. However, it is important that the trading regions identified for *either* imports or exports are mutually exclusive. For example, it is not permissible to regionally disaggregate imports across both the Southern African Development Community (SADC) and the Common Market for Eastern and Southern Africa (COMESA), since there are countries that are members of both trading regions. However, it is possible for example to include SADC as an export region and COMESA as an import region.

In describing the adjustments to the IFPRI model, the equation numbers refer to those found in Lofgren *et al* (2002) and equation letters refer to those found in the South African model. Equations 1 and 2 in the IFPRI model are now replaced with Equations A and B respectively. The difference between the two models is that these equations now refer to only those traded commodities that do not have regionally disaggregated trade data (i.e. *CMNR* and *CENR*).

$$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'c} \qquad c \in CMNR \qquad \text{A}$$

$$\begin{bmatrix} import \\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} import \\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} tariff \\ adjust- \\ ment \end{bmatrix} \cdot \begin{bmatrix} exchange\ rate \\ (LCU\ per \\ FCU) \end{bmatrix} + \begin{bmatrix} cost\ of\ trade \\ inputs\ per \\ import\ unit \end{bmatrix}$$

$$PE_c = pwe_c \cdot EXR - \sum_{c' \in CT} PQ_c \cdot ice_{c'c} \qquad c \in CENR \qquad \text{B}$$

$$\begin{bmatrix} aggregate \\ export\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} world\ export \\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} exchange\ rate \\ (LCU\ per \\ FCU) \end{bmatrix} - \begin{bmatrix} cost\ of\ trade \\ inputs\ per \\ export\ unit \end{bmatrix}$$

Three new equations are added to the model to allow for the regional disaggregation of exports. For those exported commodities that are regionally disaggregated the equation for the regional export price ($PER_{c\,r}$) is given in Equation D. Note that $PER_{c\,r}$ is analogous to PE_c except in its inclusion of a regional subscript. Under the small-country assumption, the regional price of an exported commodity is equal to that commodity's world export price ($pwer_{c\,r}$) times the exchange rate (EXR). Furthermore, since the export price represents the amount received by producers per unit sold abroad, the transaction costs per unit of output are removed from this price. This is equal to the share of transaction costs per commodity unit ($icer_{c'c\,r}$) multiplied by the market price at which these transaction commodities are sold (PQ_c).

Regional export prices and quantities ($QER_{c\,r}$) are combined under a CES function to arrive at an aggregate export price (PE_c) and quantity (QE_c). This aggregation is shown in Equations E and F. The ease at which exports can shift between regions is governed by the elasticity of substitution, which is a transformation of ρ_c^e.

$$PER_{c\,r} = pwer_{c\,r} \cdot EXR - \sum_{c' \in CT} PQ_c \cdot icer_{c'c\,r} \qquad \begin{matrix} c \in CER \\ r \in R \end{matrix} \qquad \text{D}$$

$$\begin{bmatrix} regional\ export \\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} regional\ export \\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} exchange\ rate \\ (LCU\ per \\ FCU) \end{bmatrix} - \begin{bmatrix} regional\ cost\ of \\ trade\ inputs\ per \\ export\ unit \end{bmatrix}$$

$$QE_c = \alpha_c^e \cdot \left(\sum_{r \in R} \delta_{c\,r}^e \cdot \left(QER_{c\,r} \right)^{-\rho_c^e} \right)^{-\frac{1}{\rho_c^e}} \qquad \begin{matrix} c \in CER \\ r \in R \end{matrix} \qquad \text{E}$$

$$\begin{bmatrix} aggregate \\ export \\ quantity \end{bmatrix} = CES \begin{bmatrix} regional \\ export \\ quantity \end{bmatrix}$$

$$\frac{PER_{c\,r}}{PE_c} = QER_{c\,r} \cdot \left(\sum_{r' \in R} \delta_{c\,r'}^e \cdot \left(QER_{c\,r'} \right)^{-\rho_c^e} \right)^{-1} \cdot \delta_{c\,r}^e \cdot \left(QER_{c\,r} \right)^{-\rho_c^e - 1} \qquad \begin{matrix} c \in CER \\ r \in R \\ r' \in R \end{matrix} \qquad \text{F}$$

$$\begin{bmatrix} regional\ to \\ aggregate\ export \\ price\ ratio \end{bmatrix} = f \begin{bmatrix} regional\ to \\ aggregate\ export \\ supply\ ratio \end{bmatrix}$$

Three new equations are also added to the model to allow for the regional disaggregation of imports. In Equation G, the price of a regionally imported commodity ($PMR_{c\,r}$) is equal to the commodity's world import price ($pwmr_{c\,r}$) multiplied by the exchange rate (EXR) and any region-specific import tariffs ($tmr_{c\,r}$). Any additional transactions costs are added, and are equal to the share of these costs per commodity unit ($icmr_{c'c\,r}$) multiplied by the market price at which these transaction commodities are sold (PQ_c).

Regional import prices and quantities ($QMR_{c\,r}$) are combined under a CES function to arrive at an aggregate import price (PM_c) and quantity (QM_c). This aggregation is shown in Equations H and I. The ease at which exports can shift between regions is governed by the elasticity of substitution, which is a transformation of ρ_c^m .

$$PMR_{c\,r} = pwmr_{c\,r} \cdot \left(1 + tmr_{c\,r} \right) \cdot EXR - \sum_{c' \in CT} PQ_c \cdot icmr_{c'c\,r} \qquad \begin{matrix} c \in CMR \\ r \in R \end{matrix} \qquad \text{G}$$

$$\begin{bmatrix} regional\ import \\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} regional\ import \\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} exchange\ rate \\ (LCU\ per \\ FCU) \end{bmatrix} - \begin{bmatrix} regional\ cost\ of \\ trade\ inputs\ per \\ import\ unit \end{bmatrix}$$

$$QM_c = \alpha_c^m \cdot \left(\sum_{r \in R} \delta_{c\,r}^m \cdot \left(QMR_{c\,r} \right)^{-\rho_c^m} \right)^{-\frac{1}{\rho_c^m}} \qquad \begin{matrix} c \in CMR \\ r \in R \end{matrix} \qquad \text{H}$$

$$\begin{bmatrix} aggregate \\ import \\ quantity \end{bmatrix} = CES \begin{bmatrix} regional \\ import \\ quantity \end{bmatrix}$$

$$\frac{PMR_{c\,r}}{PM_c} = QMR_{c\,r} \cdot \left(\sum_{r' \in R'} \delta_{c\,r'}^m \cdot \left(QMR_{c\,r'} \right)^{-\rho_c^m} \right)^{-1} \cdot \delta_{c\,r}^m \cdot \left(QMR_{c\,r} \right)^{-\rho_c^m - 1} \qquad \begin{matrix} c \in CMR \\ r \in R \\ r' \in R \end{matrix} \qquad \text{I}$$

$$\begin{bmatrix} regional\ to \\ aggregate\ import \\ price\ ratio \end{bmatrix} = f \begin{bmatrix} regional\ to \\ aggregate\ import \\ supply\ ratio \end{bmatrix}$$

Since tariff revenue and import earnings are now disaggregated across regions for some commodities, it is also necessary to adjust the government income and current account equations in the IFPRI model. Equations 37 and 41 in the IFPRI model are replaced with Equations J and K below.

$$YG = \sum_{i \in INSDNG} tins_i \cdot YI_i + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a +$$
$$\sum_{c \in CMNR} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{r \in R} \sum_{c \in CMR} tmr_{cr} \cdot pwmr_{cr} \cdot QMR_{cr} \cdot EXR +$$
$$\sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YF_{gov\ f} + trnsfr_{gov\ row} \cdot EXR \qquad \text{J}$$
$$\begin{bmatrix} government \\ revenue \end{bmatrix} = \begin{bmatrix} direct\ taxes \\ from \\ institutions \end{bmatrix} + \begin{bmatrix} activity \\ tax \end{bmatrix} + \begin{bmatrix} import \\ tariffs \end{bmatrix} + \begin{bmatrix} sales \\ tax \end{bmatrix} + \begin{bmatrix} factor \\ income \end{bmatrix} + \begin{bmatrix} transfers \\ from \\ RoW \end{bmatrix}$$

$$\sum_{c \in CMNR} pwm_c \cdot QM_c + \sum_{r \in R} \sum_{c \in CMR} pwmr_{cr} \cdot QMR_{cr} \cdot \sum_{f \in F} trnsfr_{row\ f}$$
$$= \sum_{c \in CENR} pwe_c \cdot QE_c + \sum_{r \in R} \sum_{c \in CER} pwer_{cr} \cdot QER_{cr} + \sum_{i \in INSD} trnsfr_{i\ row} + FSAV \qquad \text{K}$$
$$\begin{bmatrix} import \\ spending \end{bmatrix} + \begin{bmatrix} factor \\ transfers \\ to\ RoW \end{bmatrix} = \begin{bmatrix} export \\ revenue \end{bmatrix} + \begin{bmatrix} institutional \\ transfers \\ from\ RoW \end{bmatrix} + \begin{bmatrix} foreign \\ savings \end{bmatrix}$$

Upward-Sloping Factor Supply Curve

Two new equations are included in the model to allow for a factor closure in which both supply and real wages are endogenously determined. Equation L allows factor supply to adjust from its original level (QFS_f^0) according to changes in the real average wage (RWF_f), with its responsiveness being governed by the wage elasticity of factor supply ($etals_f$). The real average wage is defined in Equation M.

$$QFS_f \big/ QFS_f^0 = \left(RWF_f \big/ RWF_f^0 \right)^{etals_f} \qquad f \in F \qquad \text{L}$$
$$\begin{bmatrix} factor\ f \\ supply\ ratio \end{bmatrix} = f \begin{bmatrix} real\ wage \\ ratio \end{bmatrix}$$

$$RWF_f = \left(\frac{YF_f}{QFS_f} \right) \Big/ \left(\frac{CPI}{CPI^0} \right) \qquad f \in F \qquad \text{M}$$
$$\begin{bmatrix} average\ real\ wage \\ per\ factor\ unit \end{bmatrix} = \begin{bmatrix} average\ wage \\ per\ factor\ unit \end{bmatrix} / \begin{bmatrix} consumer\ price \\ index\ ratio \end{bmatrix}$$

Factor-Specific Productivity

Equations 15 and 16 in the IFPRI model are replaced by Equations N and O below. The only difference between the equations is the inclusion below of a factor-specific productivity adjustment term ($\alpha^{vaf}_{f\,a}$). In the initial equilibrium or base year the value of this term is set one.

$$QVA_a = \alpha^{va}_a \cdot \left(\sum_{f \in F} \delta^{va}_{f\,a} \cdot \left(\alpha^{vaf}_{f\,a} \cdot QF_{f\,a} \right)^{-\rho^{va}_a} \right)^{-\frac{1}{\rho^{va}_a}} \qquad a \in A \qquad \text{N}$$

$$\begin{bmatrix} \textit{quantity of aggregate} \\ \textit{value-added} \end{bmatrix} = CES \begin{bmatrix} \textit{factor} \\ \textit{inputs} \end{bmatrix}$$

$$W_f \cdot WFDIST_{fa} = PVA_a \cdot \left(1 - tva_a\right) \cdot QVA_a \cdot \left(\sum_{f \in F'} \delta^{va}_{f\,a} \cdot \left(\alpha^{vaf}_{f\,a} \cdot QF_{f\,a} \right)^{-\rho^{va}_a} \right)^{-1} \cdot$$

$$\delta^{va}_{f\,a} \cdot \left(\alpha^{vaf}_{f\,a} \cdot QF_{f\,a} \right)^{-\rho^{va}_a - 1} \qquad \begin{matrix} a \in A \\ f \in F \end{matrix} \qquad \text{O}$$

$$\begin{bmatrix} \textit{marginal cost of} \\ \textit{factor f in activity a} \end{bmatrix} = \begin{bmatrix} \textit{marginal revenue product} \\ \textit{of factor f in activity a} \end{bmatrix}$$

3.2 Dynamic Model Specification

Section 2.1 described the within-period or static component of the South African CGE model. However, the impact of policy-changes includes dynamic aspects, such as the inter-temporal effects of changes in investment and the rate of capital accumulation. In order to investigate in more detail the relationship between policy-changes, factor accumulation, and productivity changes, the static model is extended to a dynamic recursive model. The static model is solved as a series of equilibriums, each one representing a distinct period, typically a single year.

Over the time period being analysed a number of policy-independent changes are assumed to take place. Together these effects form a projected or counterfactual growth path for the economy. These inter-period adjustments include population and labour force growth, capital accumulation, factor productivity changes, and changes in government expenditure. This section describes the dynamic extensions of the static model with reference to the mathematical equations presented in the previous section and Lofgren *et al* (2002). This is done for each of the inter-period adjustments.

Population Growth

As described Section 2.1, each representative household consumes commodities under a Linear Expenditure System (LES) of demand. Equation 33 from the IFPRI model is shown below. This system allows for an income-independent level of consumption ($PQ_c \cdot \gamma_{ch}^m$) measured as the market value of each household's consumption of each commodity that is unaffected by changes in disposable income. The remaining terms in Equation 33 determine the level of additional consumption demand that adjusts with changes in income.

$$PQ_c \cdot QH_{c\,h} = PQ_c \cdot \gamma_{ch}^m + \beta_{ch}^m \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma_{c'h}^m \right) \qquad \begin{matrix} c \in C \\ h \in H \end{matrix} \qquad 33$$

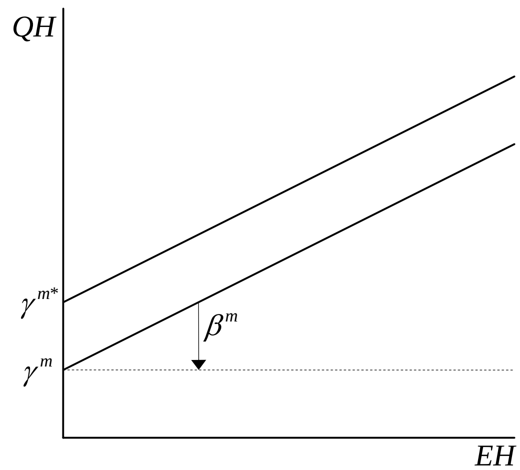
$$\begin{bmatrix} \textit{household consumption} \\ \textit{spending on market} \\ \textit{commodity } c \end{bmatrix} = f \begin{bmatrix} \textit{total household consumption} \\ \textit{spending, market price of c, other} \\ \textit{commodity prices (market and home)} \end{bmatrix}$$

Population growth is assumed to enter the model through its direct and positive affect on the level of private consumption spending. During the dynamic updating process and as the population grows, the level of each household's consumption of a particular commodity is adjusted upwards to account for greater consumption demand. This is achieved by increasing the quantity of income-independent demand (γ_{ch}^m) at the same rate as population growth.

Equation 33 is represented graphically in Figure A.1 for a single representative household's consumption of a particular commodity ($QH_{c\,h}$). This is then related to the household's level of total consumption spending (EH_h). The upward-sloping consumption demand curve reflects the positive relationship between the household's disposable income and the level of consumption. Initially the level of income-independent consumption is given by γ^m. Under the LES specification there is a linear relationship between income and consumption, and this is reflected in the constant slope (β^m) of the consumption curve.

In the dynamic model, population growth increases the value of γ^m proportionately and causes the consumption curve to shift upwards to reflect the higher level of minimum consumption (γ^{m*}). As seen in the figure, it is assumed that the slope of the consumption curve (β^m) remains unchanged. Therefore population growth is assumed to affect only average, and not marginal, consumption demand. Accordingly, new consumers are assumed to share the same consumption preferences as existing consumers.

Figure A.1: Household Consumption Demand and Population Growth



Labour Force Growth

The method of updating the relevant parameters to reflect changes in labour supply in the current model depends on the labour market closure adopted for each labour category. Four alternative closure options are possible for each factor market. In the first case, labour supply is flexible but constrained in its ability to adjust by the real wage elasticity of labour supply. No exogenous updating of labour supply (QFS_f) is necessary, since labour supply adjusts endogenously to determine final employment and wages.[10] However, if labour supply for this factor is growing exogenously then QFS_f^0 in Equation L is adjusted accordingly. In the second closure option, sectoral demand for a labour category is held fixed, and any adjustments in demand following changes in labour supply are exogenous. In this case it is assumed that growth in supply is the same across all sectors. In the third closure option, labour is assumed to be unemployed at a fixed real wage. This represents a special case of the first closure option when the wage elasticity of labour supply ($etals_f$) is infinity. Therefore the exogenous adjustment of labour supply (QFS_f) is unnecessary since there are no constraints on factor supply. Rather it is necessary to exogenously adjust real wages. The fourth closure option assumes that factor supply is fixed and the real wage adjusts to equate demand and supply. This final closure implies full employment. Between-periods

[10] As seen in Equations L and M, changes in labour supply and real wages are dependent on these variables' previous-period values. As such these values are updated between periods.

the fixed level of labour supply is adjusted exogenously. This also represents a special case of the first closure where the wage elasticity of labour supply ($etals_f$) is zero.

Capital Accumulation

Unlike labour supply, which is either determined exogenously or by market closure, all changes in total capital supply are endogenous in the dynamic model. In a given time period the total available capital is determined by the previous period's capital stock and investment spending. However, what remains to be decided is how the new capital stock resulting from previous investment is to be allocated across sectors.

An extreme specification of the model would allocate investment in proportion to each sector's share in aggregate capital income or profits. However, in the current dynamic model, these proportions are adjusted by the ratio of each sector's profit rate to the average profit rate for the economy as a whole. Sectors with a higher-than-average profit rate receive a larger share of investment than their share in aggregate profits. This updating process involves four steps.

Equation N describes the first step at which the average economy-wide rental rate of capital ($AWF^a_{f\,t}$) is calculated for time period t. This is equal to the sum of the rental rates of each sector weighted by the sector's share of total capital factor demand.[11]

$$AWF^a_{f\,t} = \sum_a \left[\left(\frac{QF_{f\,a\,t}}{\sum_{a'} QF_{f\,a'\,t}} \right) \cdot WF_{f\,t} \cdot WFDIST_{f\,a\,t} \right] \qquad \begin{matrix} f \text{ is capital} \\ a \in A \\ a' \in A \\ t \in T \end{matrix} \qquad \text{P}$$

$$\begin{bmatrix} \textit{average capital} \\ \textit{rental rate} \end{bmatrix} = \begin{bmatrix} \textit{weighted sum of sectors'} \\ \textit{capital rental rates} \end{bmatrix}$$

In the second step each sector's share of the new capital investment ($\eta^a_{f\,a\,t}$) is calculated by comparing its rental rate to the economy-wide average. For those sectors with above average rental rates, the second term on the right-hand side of Equation O will be greater than one. The converse would be true for sectors with rental rates that are below average. This term is then multiplied by the existing share of capital stock to arrive at a sectoral distribution for new capital. The inter-sectoral mobility of investment is indicated by β^a. In the extreme case where β^a is zero there is no

[11] Although there is only a single capital factor in the South African model, the subscript *f* is maintained in order to remain consistent with the notation of the static model described in Lofgren *et al* (2002).

inter-sectoral mobility of investment funds, and all investment can be thought of as being funded by retained profits.

$$\eta^{a}_{f\,a\,t}=\left(\frac{QF_{f\,a\,t}}{\sum_{a'}QF_{f\,a'\,t}}\right)\cdot\left(\beta^{a}\cdot\left(\frac{WF_{f,t}\cdot WFDIST_{f\,a\,t}}{AWF^{a}_{f\,t}}-1\right)+1\right) \qquad \begin{matrix} f \text{ is capital} \\ a\in A \\ a'\in A \\ t\in T \end{matrix} \qquad \text{Q}$$

$$\begin{bmatrix}\textit{share of}\\ \textit{new capital}\end{bmatrix}=\begin{bmatrix}\textit{share of}\\ \textit{existing capital}\end{bmatrix}\cdot\begin{bmatrix}\textit{capital rental}\\ \textit{rate ratio}\end{bmatrix}$$

Equation P shows the third step of the updating procedure in which the quantity of new capital is calculated as the value of gross fixed capital formation divided by the price of capital ($PK_{f\,t}$). This is then multiplied by each sector's share of new capital ($\eta^{a}_{f\,a\,t}$) to arrive at a final quantity allocated to each sector ($\Delta K^{a}_{f\,a\,t}$). The determination of the unit capital price is shown in Equation Q.

$$\Delta K^{a}_{f\,a\,t}=\eta^{a}_{f\,a\,t}\cdot\left(\frac{\sum_{c}PQ_{c\,t}\cdot QINV_{c\,t}}{PK_{f\,t}}\right) \qquad \begin{matrix} f \text{ is capital} \\ a\in A \\ c\in C \\ t\in T \end{matrix} \qquad \text{R}$$

$$\begin{bmatrix}\textit{quantity of new}\\ \textit{capital by sector}\end{bmatrix}=\begin{bmatrix}\textit{share of}\\ \textit{new capital}\end{bmatrix}\cdot\begin{bmatrix}\textit{total quantity of}\\ \textit{new capital}\end{bmatrix}$$

$$PK_{f\,t}=\sum_{c}PQ_{c\,t}\cdot\frac{QINV_{c\,t}}{\sum_{c'}QINV_{c'\,t}} \qquad \begin{matrix} f \text{ is capital} \\ a\in A \\ c\in C \\ c'\in C \\ t\in T \end{matrix} \qquad \text{S}$$

$$\begin{bmatrix}\textit{unit price}\\ \textit{of capital}\end{bmatrix}=\begin{bmatrix}\textit{weighted market price}\\ \textit{of investment commodities}\end{bmatrix}$$

In the final step the new aggregate quantity of capital ($QFS_{f\,t+1}$) and the sectoral quantities of capital ($QF_{f\,a\,t+1}$) are adjusted from their previous levels to include new additions to the capital stock. Over and above these changes there is also a loss of capital to account for depreciation (υ_{f}).

$$QF_{f\,a\,t+1}=QF_{f\,a\,t}\cdot\left(1+\frac{\Delta K^{a}_{f\,a\,t}}{QF_{f\,a\,t}}-\upsilon_{f}\right) \qquad \begin{matrix} f \text{ is capital} \\ a\in A \\ t\in T \end{matrix} \qquad \text{T}$$

$$\begin{bmatrix}\textit{average capital}\\ \textit{rental rate}\end{bmatrix}=\begin{bmatrix}\textit{weighted sum of sectors'}\\ \textit{capital rental rates}\end{bmatrix}$$

$$QFS_{f\,t+1}=QFS_{f\,t}\cdot\left(1+\frac{\sum_{a}\Delta K_{f\,a\,t}}{QFS_{f\,t}}-\upsilon_{f}\right) \qquad \begin{matrix} f \text{ is capital} \\ a\in A \\ t\in T \end{matrix} \qquad \text{U}$$

$$\begin{bmatrix}\textit{average capital}\\ \textit{rental rate}\end{bmatrix}=\begin{bmatrix}\textit{weighted sum of sectors'}\\ \textit{capital rental rates}\end{bmatrix}$$

The above specification of capital accumulation and allocation is not fully inter-temporal. It is assumed that any expectations that influence the level and distribution of investment are based on past experience. While this is an assumption, it does greatly simplify the dynamics of the model and avoids the specification of inter-temporal optimisation.

Total and Factor-Specific Productivity Growth

Along with changes in factor supply, the dynamic model also takes into consideration changes in factor productivity. This is done by multiplying either the α_a^{va} parameter in Equation N by the percentage change in total factor productivity (TFP), or $\delta_{f\,a}^{va}$ in the case of factor-specific productivity.

Government Consumption and Transfer Spending

Since government consumption spending and transfers to households are fixed in real terms within a particular period it is necessary to exogenously increase these payments between periods. This done by increasing the value of qg_c in Equation 36 in the IFPRI model in the case of government consumption spending, and $trnsfr_{i\,gov}$ in Equation 38 in the case of government transfers to households.

4. Applications of the Model and Areas for Further Research

This section concludes this documentation by first discussing existing and potential applications of the South African model. In each case the core model is extended to allow for the appropriate treatment of the issue being analysed. However, beyond adjusting the model to address specific policy questions, it is also necessary for the model's specification be strengthened through continued research on its various components. Some of the supporting research that would improve the model's representation of the structure of the South African economy are discussed last.

Past and Potential Applications of the South African Model

A number of studies using the South African model have already been undertaken. These cover a wide range of issues, including health and health policy; social security and public finance; and labour market and trade policies. For example, Thurlow and van Seventer (forthcoming) used the model to asses the poverty and distributional impacts of alternative bilateral free trade agreements between South Africa and its major trading partners. The model accompanying this documentation reproduces these results, which are also described in the attached paper. As an extension of the work presented in Thurlow (2002), the author uses the dynamic model to assess the macroeconomic impact of implementing and financing a basic income grant. Davies (2002) considers the effects of alternative labour market policies on future levels of employment. Finally, Thurlow (2007) assesses the impact of trade liberalisation, reform on the South African economy, with attention to engendered impacts.

Examples of other issues to which the model could be applied include: (i) the economic and welfare implications of investment and other developments within industrial sectors; (ii) the impact of broad and specific government fiscal policy on both economic performance and poverty; (iii) the economic and welfare implications of alternative government taxation schemes; and (iv) the influence of production and policy on the environment. Although the list of possible applications is far from exhaustive, it does indicate the broad scope of economy-wide modelling.

Areas for Further Supporting Research

A number of areas of the model require further research and development. Currently the model is run as a series of solutions, each one representing a single year. A better framework would allow the model to run simulations in a single solution. However, beyond the extension of the model to a single-solution framework, which is currently underway, the identification of the role of expectations in the real economy requires some attention before the model can be specified using inter-temporal optimisation dynamics. For example, more information is needed on the extent to which the investment allocation decision in South Africa is governed by forward-looking expectations rather than adaptive behaviour.

Currently the model employs a CES neoclassical production structure with constant returns to scale. More sectoral-level research that validates either the current specification or a more appropriate production structure would greatly improve the model's representation of the specific workings of the South African economy. For example, the model assumes that low-skilled and high-skilled labour is equally substitutable for capital. This is clearly an abstraction for the real workings of the factor markets. Beyond extending the model to include a more appropriate and flexible factor substitution function, which is currently underway, research is needed that estimates the parameters that would calibrate this new specification. The estimation of sectoral production functions would also cast light on the importance of scale economies within each sector, and the importance of excess capacity in production and labour demand. Information is also needed on the degree of factor mobility between sectors, and on the wage elasticity of labour supply.

Finally, the disaggregation of the public sector into the various functions of government would greatly improve the analysis of government policies. Similarly, the gradual inclusion of financial markets into the model would broaden the range of policy questions that could be addressed.

Appendix A: Model Specification

The following tables provide a complete listing of the model's variables, parameters and equations. Although these tables describe the South African model, it is largely based on the equation listing found in Lofgren *et al* (2002). However, the equation numbers do not correspond to those found in Lofgren *et al* (2002). Rather the ordering of equations follows the description of the model found in Section 2 of this paper.

Table A1: Model Sets, Parameters, and Variables

Symbol	Explanation	Symbol	Explanation
Sets			
$a \in A$	Activities	$c \in CMR(\subset C)$	Regionally imported commodities
$a \in ALEO(\subset A)$	Activities with a Leontief function at the top of the technology nest	$c \in CMNR(\subset C)$	Non-regionally imported commodities
$c \in C$	Commodities	$c \in CT(\subset C)$	Transaction service commodities
$c \in CD(\subset C)$	Commodities with domestic sales of domestic output	$c \in CX(\subset C)$	Commodities with domestic production
$c \in CDN(\subset C)$	Commodities not in *CD*	$f \in F$	Factors
$c \in CE(\subset C)$	Exported commodities	$i \in INS$	Institutions (domestic and rest of world)
$c \in CEN(\subset C)$	Commodities not in *CE*	$i \in INSD(\subset INS)$	Domestic institutions
$c \in CM(\subset C)$	Aggregate imported commodities	$i \in INSDNG(\subset INSD)$	Domestic non-government institutions
$c \in CMN(\subset C)$	Commodities not in *CM*	$h \in H(\subset INSDNG)$	Households
Parameters			
$cwts_c$	Weight of commodity c in the CPI	pwm_c	Import price (foreign currency)
$dwts_c$	Weight of commodity c in the producer price index	$pwmr_{cr}$	Import price by region (foreign currency)
ica_{ca}	Quantity of c as intermediate input per unit of activity a	$qdst_c$	Quantity of stock change
$icd_{cc'}$	Quantity of commodity c as trade input per unit of c' produced and sold domestically	$\overline{qg}_c$	Base-year quantity of government demand
$ice_{cc'}$	Quantity of commodity c as trade input per exported unit of c'	$\overline{qinv}_c$	Base-year quantity of private investment demand
$icer_{cc'r}$	Quantity of commodity c as trade input per exported unit of c' from region r	$shif_{if}$	Share for domestic institution i in income of factor f
$icm_{cc'}$	Quantity of commodity c as trade input per imported unit of c'	$shii_{ii'}$	Share of net income of i' to i (i' ∈ INSDNG'; i ∈ INSDNG)
$icmr_{cc'r}$	Quantity of commodity c as trade input per imported unit of c' from region r	ta_a	Tax rate for activity a
$inta_a$	Quantity of aggregate intermediate input per activity unit	$\overline{tins}_i$	Exogenous direct tax rate for domestic institution i
iva_a	Quantity of aggregate intermediate input per activity unit	$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$\overline{mps}_i$	Base savings rate for domestic institution i	tm_c	Import tariff rate
$mps01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates	tmr_{cr}	Regional import tariff
pwe_c	Export price (foreign currency)	tq_c	Rate of sales tax
$pwer_{cr}$	Export price by region (foreign currency)	$trnsfr_{if}$	Transfer from factor f to institution i

Source: South African Model and Lofgren *et al* (2002).

Table A1 continued: Model Sets, Parameters, and Variables

Symbol	Explanation	Symbol	Explanation
Greek Symbols			
α_a^a	Efficiency parameter in the CES activity function	δ_c^t	CET function share parameter
α_a^{va}	Efficiency parameter in the CES value-added function	δ_{fa}^{va}	CES value-added function share parameter for factor *f* in activity *a*
α_c^{ac}	Shift parameter for domestic commodity aggregation function	γ_{ch}^m	Subsistence consumption of marketed commodity c for household h
α_c^q	Armington function shift parameter	θ_{ac}	Yield of output c per unit of activity a
α_c^t	CET function shift parameter	ρ_a^a	CES production function exponent
α_c^m	Shift parameter in the CES regional import function	ρ_a^{va}	CES value-added function exponent
α_c^e	Shift parameter in the CES regional export function	ρ_c^{ac}	Domestic commodity aggregation function exponent
β^a	Capital sectoral mobility factor	ρ_c^q	Armington function exponent
β_{ch}^m	Marginal share of consumption spending on marketed commodity c for household h	ρ_c^t	CET function exponent
δ_a^a	CES activity function share parameter	ρ_c^m	Regional imports aggregation function exponent
δ_{ac}^{ac}	Share parameter for domestic commodity aggregation function	ρ_c^e	Regional exports aggregation function exponent
δ_c^q	Armington function share parameter	η_{fat}^a	Sector share of new capital
υ_f	Capital depreciation rate		
Exogenous Variables			
$\overline{CPI}$	Consumer price index	$\overline{MPSADJ}$	Savings rate scaling factor (= 0 for base)
$\overline{DTINS}$	Change in domestic institution tax share (= 0 for base; exogenous variable)	$\overline{QFS}_f$	Quantity supplied of factor
$\overline{FSAV}$	Foreign savings (FCU)	$\overline{TINSADJ}$	Direct tax scaling factor (= 0 for base; exogenous variable)
$\overline{GADJ}$	Government consumption adjustment factor	$\overline{WFDIST}_{fa}$	Wage distortion factor for factor f in activity a
$\overline{IADJ}$	Investment adjustment factor		
Endogenous Variables			
AWF_{ft}^a	Average capital rental rate in time period t	QF_{fa}	Quantity demanded of factor f from activity a
$DMPS$	Change in domestic institution savings rates (= 0 for base; exogenous variable)	QG_c	Government consumption demand for commodity
DPI	Producer price index for domestically marketed output	QH_{ch}	Quantity consumed of commodity c by household h
EG	Government expenditures	QHA_{ach}	Quantity of household home consumption of commodity c from activity a for household h
EH_h	Consumption spending for household	$QINTA_a$	Quantity of aggregate intermediate input
EXR	Exchange rate (LCU per unit of FCU)	$QINT_{ca}$	Quantity of commodity c as intermediate input to activity a
$GOVSHR$	Government consumption share in nominal absorption	$QINV_c$	Quantity of investment demand for commodity
$GSAV$	Government savings	QM_c	Quantity of imports of commodity c
$INVSHR$	Investment share in nominal absorption	QMR_{cr}	Quantity of imports of commodity c by region r

Source: South African Model and Lofgren *et al* (2002).

Table A1 concluded: Model Sets, Parameters, and Variables

Symbol	Explanation	Symbol	Explanation
Endogenous Variables Continued			
MPS_i	Marginal propensity to save for domestic non-government institution (exogenous variable)	QER_{cr}	Quantity of exports of commodity c to region r
PA_a	Activity price (unit gross revenue)	QQ_c	Quantity of goods supplied to domestic market (composite supply)
PDD_c	Demand price for commodity produced and sold domestically	QT_c	Quantity of commodity demanded as trade input
PDS_c	Supply price for commodity produced and sold domestically	QVA_a	Quantity of (aggregate) value-added
PE_c	Export price (domestic currency)	QX_c	Aggregated quantity of domestic output of commodity
PER_{cr}	Export price by region (domestic currency)	$QXAC_{ac}$	Quantity of output of commodity c from activity a
$PINTA_a$	Aggregate intermediate input price for activity a	RWF_f	Real average factor price
PK_{ft}	Unit price of capital in time period t	$TABS$	Total nominal absorption
PM_c	Import price (domestic currency)	$TINS_i$	Direct tax rate for institution i ($i \in$ INSDNG)
PMR_{cr}	Import price by region (domestic currency)	$TRII_{ii'}$	Transfers from institution i' to i (both in the set INSDNG)
PQ_c	Composite commodity price	WF_f	Average price of factor
PVA_a	Value-added price (factor income per unit of activity)	YF_f	Income of factor f
PX_c	Aggregate producer price for commodity	YG	Government revenue
$PXAC_{ac}$	Producer price of commodity c for activity a	YI_i	Income of domestic non-government institution
QA_a	Quantity (level) of activity	YIF_{if}	Income to domestic institution i from factor f
QD_c	Quantity sold domestically of domestic output	ΔK^{a}_{fat}	Quantity of new capital by activity a for time period t
QE_c	Quantity of exports		

Source: South African Model and Lofgren *et al* (2002).

Table A2: Model Equations

Production and Price Equations	
$QINT_{c\,a} = ica_{c\,a} \cdot QINTA_a$	(1)
$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{ca}$	(2)
$QVA_a = \alpha_a^{va} \cdot \left(\sum_{f \in F} \delta_{f\,a}^{va} \cdot \left(\alpha_{f\,a}^{vaf} \cdot QF_{f\,a} \right)^{-\rho_a^{va}} \right)^{-\frac{1}{\rho_a^{va}}}$	(3)
$W_f \cdot \overline{WFDIST}_{fa} = PVA_a \cdot (1 - tva_a) \cdot QVA_a \cdot \left(\sum_{f \in F'} \delta_{f\,a}^{va} \cdot \left(\alpha_{f\,a}^{vaf} \cdot QF_{f\,a} \right)^{-\rho_a^{va}} \right)^{-1} \cdot \delta_{f\,a}^{va} \cdot \left(\alpha_{f\,a}^{vaf} \cdot QF_{f\,a} \right)^{-\rho_a^{va}-1}$	(4)
$QVA_a = iva_a \cdot QA_a$	(5)
$QINTA_a = inta_a \cdot QA_a$	(6)
$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a$	(7)
$QXAC_{a\,c} = \theta_{a\,c} \cdot QA_a$	(8)
$PA_a = \sum_{c \in C} PXAC_{ac} \cdot \theta_{ac}$	(9)
$QX_c = \alpha_c^{ac} \cdot \left(\sum_{a \in A} \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_c^{ac}} \right)^{-\frac{1}{\rho_c^{ac}-1}}$	(10)
$PXAC_{a\,c} = PX_c \cdot QX_c \left(\sum_{a \in A'} \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_c^{ac}} \right)^{-1} \cdot \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_c^{ac}-1}$	(11)
$PER_{c\,r} = pwer_{c\,r} \cdot EXR - \sum_{c' \in CT} PQ_c \cdot icer_{c'c\,r}$	(12)
$QE_c = \alpha_c^e \cdot \left(\sum_{r \in R} \delta_{c\,r}^e \cdot \left(QER_{c\,r} \right)^{-\rho_c^e} \right)^{-\frac{1}{\rho_c^e}}$	(13)
$\frac{PER_{c\,r}}{PE_c} = QER_{c\,r} \cdot \left(\sum_{r' \in R} \delta_{c\,r'}^e \cdot \left(QER_{c\,r'} \right)^{-\rho_c^e} \right)^{-1} \cdot \delta_{c\,r}^e \cdot \left(QER_{c\,r} \right)^{-\rho_c^e-1}$	(14)
$PE_c = pwe_c \cdot EXR - \sum_{c' \in CT} PQ_c \cdot ice_{c'c}$	(15)
$QX_c = \alpha_c^t \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^t} + (1 - \delta_c^t) \cdot QD_c^{\rho_c^t} \right)^{\frac{1}{\rho_c^t}}$	(16)
$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \cdot \frac{1-\delta_c^t}{\delta_c^t} \right)^{\frac{1}{\rho_c^t-1}}$	(17)
$QX_c = QD_c + QE_c$	(18)
$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$	(19)
$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'c}$	(20)

Source: South African Model and Lofgren *et al* (2002).

Table A2 continued: Model Equations

Production and Price Equations Continued	
$PMR_{cr} = pwmr_{cr} \cdot \left(1+tmr_{cr}\right) \cdot EXR - \sum_{c' \in CT} PQ_{c} \cdot icmr_{c'cr}$	(21)
$QM_{c} = \alpha_{c}^{m} \cdot \left(\sum_{r \in R} \delta_{cr}^{m} \cdot \left(QMR_{cr}\right)^{-\rho_{c}^{m}} \right)^{-\frac{1}{\rho_{c}^{m}}}$	(22)
$\frac{PMR_{cr}}{PM_{c}} = QMR_{cr} \cdot \left(\sum_{r' \in R'} \delta_{cr'}^{m} \cdot \left(QMR_{cr'}\right)^{-\rho_{c}^{m}} \right)^{-1} \cdot \delta_{cr}^{m} \cdot \left(QMR_{cr}\right)^{-\rho_{c}^{m}-1}$	(23)
$PM_{c} = pwm_{c} \cdot \left(1+tm_{c} \right) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'c}$	(24)
$QQ_{c} = \alpha_{c}^{q} \cdot \left(\delta_{c}^{q} \cdot QM_{c}^{-\rho_{c}^{q}} + (1-\delta_{c}^{q}) \cdot QD_{c}^{-\rho_{c}^{q}} \right)^{-\frac{1}{\rho_{c}^{q}}}$	(25)
$\frac{QM_{c}}{QD_{c}} = \left(\frac{PDD_{c}}{PM_{c}} \cdot \frac{\delta_{c}^{q}}{1-\delta_{c}^{q}} \right)^{\frac{1}{1+\rho_{c}^{q}}}$	(26)
$QQ_{c} = QD_{c} + QM_{c}$	(27)
$PQ_{c} \cdot \left(1-tq_{c} \right) \cdot QQ_{c} = PDD_{c} \cdot QD_{c} + PM_{c} \cdot QM_{c}$	(28)
$QT_{c} = \sum_{c' \in C'} \left(icm_{cc'} \cdot QM_{c'} + icmr_{cc'} \cdot QMR_{c'} + ice_{cc'} \cdot QE_{c'} + icer_{cc'} \cdot QER_{c'} + icd_{cc'} \cdot QD_{c'} \right)$	(29)
$\overline{CPI} = \sum_{c \in C} PQ_{c} \cdot cwts_{c}$	(30)
$DPI = \sum_{c \in C} PDS_{c} \cdot dwts_{c}$	(31)
Institutional Incomes and Domestic Demand Equations	
$YF_{f} = \sum_{a \in A} WF_{f} \cdot \overline{WFDIST}_{fa} \cdot QF_{fa}$	(32)
$YIF_{if} = shif_{if} \cdot \left[YF_{f} - trnsfr_{row\,f} \cdot EXR \right]$	(33)
$YI_{i} = \sum_{f \in F} YIF_{if} + \sum_{i' \in INSDNG'} TRII_{ii'} + trnsfr_{i\,gov} \cdot \overline{CPI} + trnsfr_{i\,row} \cdot EXR$	(34)
$TRII_{ii'} = shii_{ii'} \cdot (1-MPS_{i'}) \cdot (1-\overline{tins}_{i'}) \cdot YI_{i'}$	(35)
$EH_{h} = \left(1 - \sum_{i \in INSDNG} shii_{ih} \right) \cdot \left(1-MPS_{h}\right) \cdot (1-\overline{tins}_{h}) \cdot YI_{h}$	(36)
$PQ_{c} \cdot QH_{ch} = PQ_{c} \cdot \gamma_{ch}^{m} + \beta_{ch}^{m} \cdot \left(EH_{h} - \sum_{c' \in C} PQ_{c'} \cdot \gamma_{c'h}^{m} \right)$	(37)
$QINV_{c} = IADJ \cdot \overline{qinv}_{c}$	(38)
$QG_{c} = \overline{GADJ} \cdot \overline{qg}_{c}$	(39)
$EG = \sum_{c \in C} PQ_{c} \cdot QG_{c} + \sum_{i \in INSDNG} trnsfr_{i\,gov} \cdot \overline{CPI}$	(40)

Source: South African Model and Lofgren *et al* (2002).

Table A2 concluded: Model Equations

Institutional Incomes and Domestic Demand Equations Continued	
$YG = \sum_{i \in INSDNG} \overline{tins}_i \cdot YI_i + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{c \in CMNR} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{r \in R} \sum_{c \in CMR} tmr_{cr} \cdot pwmr_{cr} \cdot QMR_{cr} \cdot EXR + \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YF_{gov\,f} + trnsfr_{gov\,row} \cdot EXR$	(41)
System Constraints and Macroeconomic Closures	
$QQ_c = \sum_{a \in A} QINT_{c\,a} + \sum_{h \in H} QH_{c\,h} + QG_c + QINV_c + qdst_c + QT_c$	(42)
$\sum_{a \in A} QF_{f\,a} = QFS_f$	(43)
$QFS_f \big/ QFS_f^0 = \left(RWF_f \big/ RWF_f^0 \right)^{etals_f}$	(44)
$RWF_f = \left(\frac{YF_f}{QFS_f} \right) \Big/ \left(\frac{CPI}{CPI^0} \right)$	(45)
$YG = EG + GSAV$	(46)
$\sum_{c \in CMNR} pwm_c \cdot QM_c + \sum_{r \in R} \sum_{c \in CMR} pwmr_{cr} \cdot QMR_{cr} \cdot \sum_{f \in F} trnsfr_{row\,f} = \sum_{c \in CENR} pwe_c \cdot QE_c + \sum_{r \in R} \sum_{c \in CER} pwer_{cr} \cdot QER_{cr} + \sum_{i \in INSD} trnsfr_{i\,row} + FSAV$	(47)
$\sum_{i \in INSDNG} MPS_i \cdot \left(1 - \overline{tins}_i\right) \cdot YI_i + GSAV + EXR \cdot FSAV = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$	(48)
$MPS_i = \overline{mps}_i \cdot \left(1 + MPSADJ\right)$	(49)
Capital Accumulation and Allocation Equations	
$AWF_{f\,t}^a = \sum_a \left[\left(\frac{QF_{f\,a\,t}}{\sum_{a'} QF_{f\,a'\,t}} \right) \cdot WF_{f\,t} \cdot WFDIST_{f\,a\,t} \right]$	(50)
$\eta_{f\,a\,t}^a = \left(\frac{QF_{f\,a\,t}}{\sum_{a'} QF_{f\,a'\,t}} \right) \cdot \left(\beta^a \cdot \left(\frac{WF_{f,t} \cdot WFDIST_{f\,a\,t}}{AWF_{f\,t}^a} - 1 \right) + 1 \right)$	(51)
$\Delta K_{f\,a\,t}^a = \eta_{f\,a\,t}^a \cdot \left(\frac{\sum_c PQ_{c\,t} \cdot QINV_{c\,t}}{PK_{f\,t}} \right)$	(52)
$PK_{f\,t} = \sum_c PQ_{c\,t} \cdot \frac{QINV_{c\,t}}{\sum_{c'} QINV_{c'\,t}}$	(53)
$QF_{f\,a\,t+1} = QF_{f\,a\,t} \cdot \left(1 + \frac{\Delta K_{f\,a\,t}^a}{QF_{f\,a\,t}} - \upsilon_f \right)$	(54)
$QFS_{f\,t+1} = QFS_{f\,t} \cdot \left(1 + \frac{\sum_a \Delta K_{f\,a\,t}}{QFS_{f\,t}} - \upsilon_f \right)$	(55)

Source: South African Model and Lofgren *et al* (2002).

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