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The Macroeconomics of Job-creating Growth

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ABSTRACT

This paper develops a theoretical macro-economic model that links social infrastructure investment, taxation, and wages to income determination and job creation. The framework incorporates productivity effects, a fiscal budget constraint, and the public good nature of social infrastructure investment and wages, identifying a multiple equilibrium problem with the possibility of a low social infrastructure investment trap. Three major results follow from the analysis.

First, fiscal austerity (characterised by reduced social infrastructure investment, lower taxes, and a low fiscal deficit) may reduce long run national income and economic capacity if the economy is in a low social infrastructure investment trap. The conventional trade-off between equity and growth disappears, and increases in social infrastructure investment and a relaxed budget constraint may improve both national income and distribution.

Second, wages play an important role in characterising the low social infrastructure investment trap and providing the government with policy alternatives. A low wage trap reinforces the scarce social infrastructure investment equilibrium. In an economy with massive unemployment, wages can provide important externalities, particularly through remittances and social inclusion effects. Firms may have insufficient incentives to raise wages to the socially optimal level, and this reluctance is reinforced by low levels of labour productivity associated with the scarcity of social capital.

Third, the low social infrastructure investment trap is reinforced by technology characterised by rapidly diminishing returns to labour. The more inelastic is the substitutability of labour for capital, the more likely will labour productivity enhancements lead to job destruction rather than job creation.

South Africa’s unemployment problem exhibits many of the characteristics associated with the low social infrastructure investment trap. Policies that may address this problem include increased taxes and borrowing to finance expanded social infrastructure investment, higher wages for the working poor, and restructuring industrial policy towards more labour-intensive production. Labour-intensive production need not entail low wage activities—industrial policy that raises labour productivity while increasing the elasticity of substitution between capital and labour can increase labour intensity while improving wages. Appropriate social infrastructure investment strategies can support this industrial policy.
1) INTRODUCTION

South Africa’s recent experience with formal sector job losses raises serious questions about the notion that economic growth, capital investment, wage restraint, and improved labour productivity are sufficient to generate job creation. Mr. Tito Mboweni’s statement in his speech at the Reserve Bank’s August general meeting of shareholders hints at jobless growth: “The modest economic growth over the past eighteen months did not lead to meaningful employment creation.” Likewise, South Africa’s rising capital-to-labour ratio demonstrates that investment can replace rather than create jobs. Even wage restraint and improved labour productivity cannot ensure adequate job creation given South Africa’s weak substitutability of labour for capital.

This paper builds a theoretical model that explains how low wages and poor rates of social infrastructure investment can create a severe trap, contributing to low employment levels and stagnant growth. The framework links the rate of job creation to investment, productivity growth, and changes in wages. The subsequent discussion appraises the relevance and implications of the model for South Africa.

Efficient social infrastructure investment, the critical policy variable in the model, can play an important role in promoting economic growth while improving distribution in countries characterised by severe inequality. Large-scale consumption-oriented redistribution may be unsustainable (and ultimately counter-productive). Yet attempts to maintain the status quo may foment severe political and social unrest, and lead to economic stagnation. Accumulated social infrastructure investment—access to high quality education, effective mechanisms for ensuring public health, economical housing integrated with efficient mass transit systems, etc.—complements labour and private capital in the production process. In addition
to increasing overall productivity, these assets improve social welfare independently of their contribution to the production of market goods and services.

Social infrastructure investment, however, must be financed, usually through taxation or budget deficits. In a closed economy, the resources come at the expense of consumption or private savings, while an open economy allows the option of external borrowing. These costs, compounded by the dead-weight loss of distortionary taxation, can undermine the positive growth effects of the social infrastructure investment. An optimal policy balances the positive growth and distributional effects of social infrastructure investment against the full economic costs imposed by taxation and/or increased indebtedness.

2) THEORETICAL MODEL

The analysis in this paper proceeds from a macroeconomic model built on microeconomic foundations. First, economic activity depends on both capital and labour, but the productivity of these factor inputs depends not only on the technology available but also on the level of social infrastructure investment and the wage rate. Greater levels of social infrastructure investment and higher wage rates increase both labour and capital productivity, although with diminishing marginal returns.

Firms maximise profits by choosing the quantity of capital and labour to employ given the policy-influenced market wage, the cost of capital, the social infrastructure investment climate, and the structure of taxation. Firms individually are too small to have a significant impact on the average wage. While raising wages collectively may improve productivity, no firm individually has sufficient incentive to increase them to the optimal level because many of the productivity benefits are diffused throughout the economy and are not appropriated exclusively by the wage-

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1 This model is mathematically developed in Appendix 2.
increasing firm. Holding a firm’s wage and tax bill constant, a firm will employ more labour and capital the greater the level of social infrastructure investment and the higher the economy’s average wage (the productivity effect). However, the higher the firm’s own tax and wage bill, holding constant social infrastructure investment and the economy’s average wage, the less the firm will employ labour and capital (the profit effect).

Both social infrastructure investment and wages improve factor effectiveness in a number of ways: (1) higher wages and social infrastructure investment contribute to improved resource distribution, reducing social tension and economising on capital inputs through fuller utilisation—fewer strikes, more opportunities for extra shifts, etc. (2) Higher wages and public investment in health and education contribute to higher labour productivity and the generation of capital-saving innovations. (3) The improved distributional effects of higher wages and increased social infrastructure investment increase expected returns to capital by reducing political risk.  

In practice, there is a link between taxes and social infrastructure investment, as well as between the representative firm’s wage bill and the average wage. At a macroeconomic level, the government weighs the trade-offs between higher taxes and greater social infrastructure investment, and output levels depend on the resulting optimising behaviour of the private sector. Government may optimally choose taxes and social infrastructure investment levels that are higher than those levels associated with maximum national income (assuming tax-financed social

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2 “Hochtief, the multi-national German construction company, may have broken off talks with Murray and Roberts, the engineering and construction group, earlier this year as a result of fears arising from the Zimbabwe crisis.... This is one of the first concrete examples of a large investment decision that was directly affected by the events in the neighbouring country.” (Business Report, September 10, 2000, page 1.)
expenditure has positive distributional benefits). Fiscal policy is developed in two stages: an overarching medium term policy rule that defines the budget constraint, and a subordinate decision with respect to tax rates and social infrastructure investment expenditure.

The model can be depicted with a graph of the feasible set of national income and social infrastructure investment outcomes associated with a given fiscal policy. Once the medium term budget constraint is specified, taxes and social infrastructure investment are jointly determined with income. This leaves the government with only one independent fiscal policy instrument. Figure 1 below presents a hypothetical economy. The dashed curve represents a non-optimal fiscal policy rule—it indicates the level of output associated with any feasible level of social infrastructure investment. The shape of the curve reflects the increasing marginal costs of taxation combined with the diminishing marginal returns to social infrastructure investment.

**FIGURE 1)**
The optimal strategy may require revising the initial fiscal policy. Mobilising the necessary resources disproportionately through taxation, for instance, may preclude borrowing for high return social infrastructure investment projects. Figure 1 depicts a hypothetical scenario in which the optimal fiscal policy offers a more favourable path from the initial point to the socially optimum point—the preferred combination of national income and social infrastructure investment.

As the economy adjusts to the social optimum, it passes the output-maximising level of social infrastructure investment. Generally, the optimal level is greater than this, since social infrastructure investment has favourable consequences for society that extend beyond its positive effects on productivity. However, at this point the government must explicitly weigh the trade-off between equity and growth.

As is apparent from the figure 1, any feasible level of national income (other than the maximum feasible) is associated with two possible levels of social infrastructure investment—one low and another high. Either of these points can represent an equilibrium. The low-level equilibrium can constitute a trap, reinforced by relatively low wages. An economy in this trap is characterised by poor social infrastructure investment outcomes, low labour productivity, high measures of inequality, and relatively low wage rates. It is possible to observe high levels of social spending in an economy in the low level trap—high but inefficient spending nonetheless translate into low levels of social infrastructure investment. Likewise, wages in the low-level equilibrium may appear relatively high compared to productivity—since low wages and low productivity mutually reinforce the trap.

The relationship between the average wage level and national income is similar to that for social infrastructure investment. For a given fiscal policy, higher wages support increasing productivity, but with diminishing returns. The feasible set
of outcomes reflects the dual impact of higher wages—as wages increase, the positive productivity effects tend to raise national income, but the negative profit effects exert negative pressure. Figure 2 below reflects this trade-off for the hypothetical economy with the assumed initial fiscal policy discussed above. At low wage levels, the marginal productivity effects are larger than the offsetting profit effects, and national income rises with the wage. As the wage increases, however, diminishing returns reduce the magnitude of subsequent marginal productivity effects, and the profit effects become stronger as the wage elasticity of labour demand increases.

**FIGURE 2)**

As is the case with social infrastructure investment, any feasible level of national income (other than the maximum) is associated with a choice between a low and a high wage rate. An economy in a low wage rate equilibrium will not necessarily reach the high wage state. If wages are market-determined, the possibility of co-ordination failure exists. Because wage increases are in part public
goods (because of resulting remittances, social cohesion, etc.), any one firm that raises its own wage pays the full cost of the increase but reaps only a fraction of the benefits. Increased productivity stemming from remittances and social cohesion accrues substantially to the larger community. The low wage trap resulting from individual firm profit-maximisation decisions is neither income-maximising nor socially optimal—and higher wages may actually increase profits. Because of the public good nature of wage increases, only co-ordination can achieve the higher wages and profits. If distributional outcomes enter into the social welfare function, the socially optimal wage will be greater than the one associated with maximum national income.

Fiscal policy affects the trade-off. If government provides inadequate social infrastructure investment, the output response of higher wages may be relatively weak. Social infrastructure investment complements wages in production—higher wages support increased productivity if the social infrastructure exists to convert income into tangible productive assets. The graph depicts the move to an optimal fiscal policy improving the trade-off between wages and national income.

3) LABOUR ABSORPTION

Economists frequently model improved productivity as increases in the marginal productivity of labour. Under assumptions of full employment, perfect competition, and profit-maximisation, this leads to commensurate increases in real wage rates, and so the labour absorption equation depends only on capital growth and the net labour-demanding impact of productivity change (which can be positive or negative). In the face of high levels of unemployment, however, labour-demanding productivity improvements lead to increased employment, and the diminishing marginal productivity of labour reduces the wage impact. Productivity
improvements may be substantially greater than those measured by wage increases. Under these conditions, the labour absorption equation can be written:

\[
\text{rate of job creation} = \text{rate of investment} + \text{capital-augmenting productivity growth} + \left( \frac{e}{c} - 1 \right) \text{labour-augmenting productivity growth} - \frac{e}{c} \text{rate of wage growth}
\]

In this equation, “e” represents the degree of substitutability between capital and labour in production (the elasticity of substitution between capital and labour) and “c” represents capital’s tax-adjusted share of income.

The ratio “e/c” provides a structural form for the wage elasticity of demand, which measures the responsiveness of job creation to changes in wages. Estimates for South Africa have found this measure to be relatively inelastic (less than one in absolute value). This implies that higher wages will increase the total wage bill (after accounting for labour market effects), and labour-augmenting productivity growth will tend to replace labour rather than increase the rate of job creation.

Increases in the rate of investment and/or capital-augmenting productivity growth will increase job creation if labour productivity and wages are constant. If capital investment embodies labour-replacing technology, however, the resulting labour-augmenting technological change may more than eliminate the resulting job creation.

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3 This equation is mathematically derived in Appendix 2, and the implications of this type of analysis are discussed in Bruton (1997).

4) IDENTIFYING THE LOW LEVEL EQUILIBRIUM

The low social infrastructure investment trap discussed above has a number of observable characteristics. The definitive diagnosis requires a well-specified structural model of factor markets and the economy’s production process, complete with fiscal interactions. However, absent this, several traits can be identified.

- A small degree of substitutability of labour for capital characterises the low social investment equilibrium. If the elasticity of substitution is relatively small, wage changes have little impact on employment and increases in labour productivity reduce the rate of job creation. At the low equilibrium, higher productivity is associated with rising capital-to-labour ratios. The Reserve Bank’s 2000 annual report documents rising labour productivity, restrained wage growth, increased capital-to-labour ratios, and falling formal sector employment. This is consistent with a low social investment equilibrium.

- An economy in the low social infrastructure investment trap is characterised by growth and employment that is relatively non-responsive to changes in the tax structure, and relatively responsive to improvements in social infrastructure investment. The opposite is true for an economy in a high social infrastructure investment equilibrium. Empirical tests of the South African economy suggest much greater sensitivity to social infrastructure investment than to tax rates.\(^5\) Tax effort analysis supports the hypothesis that South Africa has significant unutilised taxable capacity, suggesting taxes can be raised to finance social infrastructure investment

\(^5\) Analysis of endogenous growth regressions demonstrates significant and large positive effects of social infrastructure investment on growth, but substantially smaller and less significant negative effects for tax rates (Samson 1998).
with positive effects on growth and employment.\textsuperscript{6} South Africa’s recent sluggish response to tax rate reductions and incentives is consistent with this hypothesis.

- An economy in a low wage trap will demonstrate a causal link from higher wages to productivity growth, since the efficiency wage effects will be stronger for lower wages and diminish as wages increase. At the high wage equilibrium, the lines of causality run from productivity to wages. A Dresdner Bank study of South African manufacturing sectors found evidence of a positive efficiency wage effect in many industries.\textsuperscript{7} This is consistent with international experience in many low wage developing countries.\textsuperscript{8}

5) POLICY IMPLICATIONS

South Africa exhibits many of the characteristics associated with the low social infrastructure investment trap. A strategy to address this problem requires careful integration of fiscal, labour, and industrial policies. The following discussion presents several policy options.

**Fiscal policy**

This analysis suggests that fiscal policy provides an important opportunity to foster job creation. Given the existing skewed distribution of social capital in South

\textsuperscript{6} A USAID study found significant unutilised taxable capacity in South Africa (Harber 1995). Other studies corroborate this result (Samson 1996; Samson, Mac Quene, van Niekerk, and Ngqungwana 1997).


\textsuperscript{8} A recent World Bank study finds “significant efficiency wage effects” using firm-level data from Mexico (Maloney and Ribeiro 1999). Another World Bank study using an endogenous growth framework for Guatemala found similar results (Sakellariou 1995). Likewise, a study of Zimbabwean firm level data is consistent with positive efficiency wage effects (Valenchik 1997). Similarly, a study of the cement industry in Turkey finds that higher wages improve productivity by increasing technical efficiency (Saygili 1998).
Africa, the marginal productivity of efficiently allocated social infrastructure investment may be very high. South Africa’s significant unutilised taxable capacity suggests the effects of higher tax rates may be weak. Financing social infrastructure investment through higher taxes not only stimulates aggregate demand while supporting the supply response, it also may shift the degree of substitutability between capital and labour in favour of job creation. A more flexible approach to fiscal policy is consistent with recently revised policy recommendations by the International Monetary Fund (2000), which emphasise “greater flexibility in accommodating rising budget deficits...for countries with sustainable macroeconomic and external debt positions and recognised scope for more productive public spending.”

**Labour policy**

The poor in an economy characterised by a low social infrastructure investment trap face very low wage employment, if any. Low or no wages reinforce low productivity, stifling human capital accumulation. Fiscal policy reforms that promote social infrastructure investment gradually raise wages, but more slowly than that attainable with direct labour market intervention. A phased programme of labour protection, including minimum wages for the lowest paid workers, may provide the necessary co-ordination that enables an economy to escape a low wage equilibrium. These policies operate most effectively when complemented by appropriate social infrastructure investment and labour-demanding industrial policy.
Industrial policy

Industrial policy can foster job creation by not only supporting more labour-intensive production but also by nurturing technology with a high degree of substitutability of labour for capital. Labour-intensive production need not entail low wage activities—industrial policy that raises labour productivity while increasing the elasticity of substitution between capital and labour can increase labour intensity while improving wages.

6) CONCLUSIONS

This paper outlines a framework for evaluating the impact of social infrastructure investment and wages on the objectives of job creation, economic growth, and social equity. The evidence and analysis argue that South Africa exhibits characteristics of a low social infrastructure investment trap reinforced by unemployment and low wages for the working poor. One potential growth path requires substantial but prudent increases in employment-creating investments such as education, training, health care, housing, and physical infrastructure. This expenditure expands aggregate demand while increasing overall productivity. The strategy’s focus on human and social capital shifts the economy’s orientation towards a more labour intensive mode of production, helping to correct biases created by historical capital-intensive industrial policies. Furthermore, by increasing labour productivity, the strategy supports a higher wage path. Industrial policy can support this process by emphasising technology characterised by a high degree of substitutability of labour for capital. This will ensure that increased labour productivity results in a rising demand for labour and consequent job creation.
APPENDIX 1) LITERATURE REVIEW

The relationship between public capital and economic growth has long been a subject of economic debate. Earlier work focused on the relationship between public and private investment. Blejer and Khan (1984) show that public investment “crowds in” private investment, while Khan and Reinhart (1990) analyse the importance of the complementarity between public and private investment. Increasing public investment that competes with the private sector leads to a decline in private investment, but additional public investment that corrects a market failure leads to increased private investment. Easterly and Schmidt-Hebbel (1993) also find that higher public investment usually leads to greater private investment.

The endogenous growth literature examines the direct link between public expenditure and growth. Aschauer (1989) identified a powerful role for public capital in the production function, identifying a strong relationship between public investment and total factor productivity growth. Cashin (1995) examines a similar question, but explicitly develops a theoretical model that analyses the effects of public investment, transfers, and taxes on economic growth, showing how both social infrastructure and transfer payments have growth-enhancing economic effects. Munnell (1992) criticises this work on methodological grounds, and Lau and Sin (1997) find a much weaker relationship between public capital and economic growth.

The practical reasons for the link between social infrastructure investment and growth has been an important topic for policy research. A World Bank (1993) report found that social infrastructure investment was a critical ingredient to the success of the high growth East Asian economies. More recently, the World Bank (1997) reports a strong link between community involvement in local public affairs and the effectiveness with which government manages high quality schools, develops innovative day care programs and job training centres, and promotes economic
growth. This highlights the inclusiveness of the concept of social infrastructure investment—it comprises not just the physical public capital stock but also embodied human capital and assets such as community involvement and social cohesion.

The debate over social infrastructure investment encompasses a broad range of issues, including a lively controversy over the productivity of public capital, with estimates of the United States output elasticity of public capital ranging from 10% (Lau and Sin 1997) to 40% (Aschauer 1989). In addition, the need to finance social infrastructure investment through taxation, borrowing, and money creation introduces questions about how macroeconomic factors affect economic growth (Fischer 1993).

The stress on fiscal resources from financing social infrastructure investment can potentially lead to macroeconomic instability or excessively high taxation and/or borrowing, undermining economic growth. Fischer (1993) analyses the role of macroeconomic factors in determining growth, finding a strong correlation between low deficits and high rates of growth. His analysis is predicated on two premises: (1) that deficits cause “crowding out”, and/or (2) that high fiscal deficits indicate that the government is “losing control of its actions”. Levine and Renelt (1992), however, find no robust relationship between macroeconomic factors and economic growth—only investment in physical and human capital explains differences in rates of growth among countries over time.

Bruton (1997) appraises these key issues linking macroeconomic policy to growth, and he identifies an alternative framework for analysing growth. He discusses specific ways in which government policy can promote job creation by increasing the substitutability of labour for capital and fostering labour productivity growth. His earlier work (Bruton and Frank 1977) develops a mathematical framework similar to that adopted in this paper for the labour absorption equation.
APPENDIX 2) THE TECHNICAL MODEL

This appendix develops a model to explain the roles of social infrastructure investment and wages in determining income, growth, and employment through productivity effects. The analysis focuses on the public good nature of social infrastructure investment and wages, which creates the possibility of multiple equilibria. The dynamic analysis of the model provides further insight into policy options supporting improved labour absorption.

The firm’s microeconomic problem

The microeconomic analysis is based on the behaviour of a representative firm facing the problem of maximising profit ($\Pi$) subject to production ($y$) and tax policy ($t$) constraints. The problem can be represented:

\[
\begin{align*}
\text{MAX} \quad & \Pi = y - t - rk - Wl \quad \text{with respect to } k \text{ and } l \\
\text{Subject to} \quad & y = y[a(E,W)k, b(E,W)l] \\
\text{and} \quad & t = t(y, k, l) \quad \text{with } t_y, t_k, t_l \geq 0
\end{align*}
\]

where $\Pi$ is the firm’s profit, $y$ is the firm’s output, $t$ is the firm’s taxes (which include income taxes paid by its workers and investors), $k$ is the firm’s demand for private capital, $l$ is the firm’s demand for labour, $E$ is aggregate social infrastructure investment (for the whole economy), $r$ is the cost of private capital (the after-tax return to investors), and $W$ is the wage rate (the after-tax wage paid to workers). Output depends not only on the physical quantities of capital and labour employed, but also on the “effectiveness” of the production factors, represented by the coefficients $a$ (for capital) and $b$ (for labour). Factor effectiveness depends on technology (embedded in the functional form for $a$ and $b$) as well as social...
infrastructure investment and the economy’s average wage rate. Both social infrastructure investment and the wage rate possess important public good characteristics—so an individual firm is unable to fully compensate for inadequate provision of social infrastructure investment or endemic low wages.

Solving the optimisation problem yields two first order conditions:

\[ y_k(E, W, k, l) = \frac{r + tk}{1 - t} \]

\[ y_l(E, W, k, l) = \frac{W + tl}{1 - t} \]

The marginal productivity of capital and labour are equated to the respective tax adjusted factor prices. These equations implicitly (and under further restrictions explicitly\(^3\)) determine factor and output demand as functions of the factor prices (\(r\) and \(W\)), the publicly determined quantity of social infrastructure investment (\(E\)), and parameters of the tax policy. Assuming diminishing marginal productivity of inputs, factor demands are decreasing in their own tax rates and output tax rates. Output and factor demands respond positively to increases in social infrastructure investment.

**The government’s budget constraint**

Fiscal policy is governed by a policy rule (implicitly subject to a feasibility constraint) that links aggregate tax collection \(T\) to the level of social infrastructure investment \(E\). The rule can be generalised as:

\[ f(E, T) = 0 \text{ with } 0 > -f_E \geq f_T \]

This general form can handle a number of policy rules. A balanced budget rule is \(f = E - T\). If a fiscal deficit is constrained to a fixed percentage of total
revenue, then the rule can be written \( f = E - T(1 + \phi) \), where \( \phi \) is the fixed percentage. Intertemporal budget constraints can relate the present values of \( E \) and \( T \). For instance, consider a simple two period model where social infrastructure investment occurs only in the first period and is financed entirely through borrowing at interest rate \( r \), which is repaid through taxes in the second period. Then the fiscal policy rule can be written \( f(PV(E), PV(T)) = 0 \) where \( E \) and \( T \) are vectors of social infrastructure investment and taxes and the present value calculation uses the return on social investment \( \rho \) as the discount rate. The defining condition becomes \( 0 > -f_{PV(E)} \geq f_{PV(T)} \) and holds as a strict equality if \( \rho = r \). If \( \rho > r \), the condition holds as a strict inequality.

The government determines the tax schedule and fiscal policy rule, which jointly with production decisions determine total tax revenue \( T \). If fiscal policy permits public borrowing, it is equal to gross social infrastructure investment less taxes (social infrastructure investment is the only role for government in this model).

**The government’s macroeconomic problem**

The government optimises the society’s welfare function, which depends on private output \( (Y) \) and social infrastructure investment \( (E) \):

\[
U = U(Y, E) \quad \text{with} \quad U_Y \geq 0, \quad U_E \geq 0
\]

---

9 For instance, a constant elasticity production function, exogenous productivity coefficients, and a linear tax policy yield a closed form solution for \( K \) and \( L \).
Social infrastructure investment enters explicitly into the welfare function because, in addition to contributing productively to private output, it directly improves the well-being of the people.\(^\text{10}\)

The optimisation problem is constrained by the aggregate production function and the policy-determined fiscal constraint, and can be written:

\[
\text{MAX } U(Y,E) \text{ with respect to } T \text{ and } E \text{ subject to } Y = Y(E,T) \text{ and } f(E,T)=0
\]

The slope of the efficient frontier defined by the constraints can be expressed:

\[
\frac{dY}{dE} = Y_E - Y_T \frac{f_E}{f_T}
\]

At lower levels of social infrastructure investment, when the positive marginal productivity effect of social infrastructure investment dominates the negative tax effect, this derivative is positive. Eventually, the derivative becomes negative as the marginal productivity of social infrastructure investment falls and the negative impact of taxes on output becomes dominant. The effect of taxes on output is moderated or intensified by the fiscal policy constraint—the more costly is borrowing, the stronger is the second term on the right hand side. The inflection point represents the output-maximising level of social infrastructure investment, where the positive contribution of increased social infrastructure investment is exactly offset by the negative impact of required taxes (adjusted for the fiscal policy effect).

The government’s macroeconomic solution

The solution to the problem is described by the first order condition:

\(^{10}\) For instance, roads contribute to production of private output as well as the utility of leisure. Access to health care improves worker productivity as well as quality of life.
At the margin, the welfare-eroding effects of higher taxes (manifested through lower output) are balanced against the positive welfare effects of greater social infrastructure investment (which include both the direct enhancements to welfare from more social infrastructure investment, as well as the benefits of higher output).

The solution is graphically depicted in the figure below. The efficient frontier represents a menu of feasible equilibria available to policy-makers—each point on the frontier represents the highest level of output consistent with fiscal policy and the optimising behaviour firms, given the choice of the level of social infrastructure investment. If society values both output and social infrastructure investment independently, then the welfare indifference curves have negative slope and are associated with higher welfare as they shift to the right.

\[ \frac{U_Y Y_T}{f_T} = \frac{U_Y Y_E + U_E}{f_E} \]

FIGURE 3)
The solution is characterised by multiple equilibria. For example, figure 4 below indicates two equilibria associated with the level of output $Y_0$. Any feasible level of output other than the maximum level of output can be produced either with a relatively low level of social infrastructure investment and a relatively high level of private capital (the low E equilibrium in figure 4 below), or with a relatively high level of social infrastructure investment and a relatively low level of private capital (the high E equilibrium in figure 4 below).

An economy may find itself at a low social infrastructure investment equilibrium for a number of reasons, such as history, politics, or error. If the government at some point in history had valued inequality even at the expense of national income, the economy might find itself on the undesirable side of the frontier, and it would require time and resources to move to the socially desirable point on the frontier. Alternatively, since each point along the efficient frontier represents a different distribution of income, minority interests might use their political and economic power to maintain the economy at a socially undesirable point. Third, the economy might be at such an equilibrium because of a policy error: economic policy is fraught with complexity and ambiguity—the history of economic policy analysis is the history of grappling with the consequences of policy mistakes.
FIGURE 4)

Efficient frontier

social infrastructure investment

low E equilibrium

high E equilibrium

Y

Y0
The discussion above assumes exogenous factor prices. Labour policy introduces another policy instrument, enabling the government to exercise influence over another objective—the wage. Factor productivity depends in this model not only on social infrastructure investment but also on the wage level. Output is non-linearly dependent on the real wage—increasing initially while efficiency wage effects dominate the price effects, and then falling once the full efficiency wage effects are realised and factor demands respond negatively to higher wages. This creates a multiple equilibrium problem similar to that encountered with social infrastructure investment. For any feasible level of output (other than the maximum), there are two possible equilibria—a low wage equilibrium associated with low productivity and output, and a high wage equilibrium associated with high productivity and output.

**The labour absorption equation**

This section derives the model’s labour absorption equation employing a methodology similar to that developed by Bruton. Profit maximisation equates the marginal productivity of labour to the tax-adjusted wage. Assuming proportional tax rates on capital, labour, and output, this can be represented:

$$Y_L = \frac{W + tL}{1 - t_Y} = \Theta W$$  where  $$\Theta = \frac{1 + tL/W}{1 - t_Y}$$

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11 See Bruton (1997) for an accessible discussion, or Bruton and Frank (1977) for a detailed derivation of a similar formulation.
Assuming homogeneity of degree one\textsuperscript{12}, the production function can be rewritten:

\[
Y = bLg(\kappa) \quad \text{where} \quad g(\kappa) = Y(\kappa, 1) \quad \text{and} \quad \kappa = aK/bL
\]

Differentiating the production function with respect to labour \(L\) and substituting into the first order condition for labour yields:

\[
Y_L = (dY/dL) = b[g(\kappa) - \kappa g'(\kappa)] = \theta W
\]

Multiplying this expression by \(L\) yields:

\[
Y - bL\kappa g'(\kappa) = \theta WL
\]

Differentiating this with respect to time (using \(\Delta X\) to denote the differential of variable \(X\), and \(\Delta X/X\) to denote the growth rate of \(X\), and dividing through by \(Y = bLg\) yields:

\[
\frac{\Delta Y}{Y} = \left[\frac{\Delta b}{b}(g'/g)\kappa + (\Delta L/L)(g'/g)\kappa + (g''/g)\kappa \Delta \kappa + (g'/g)\kappa \Delta \kappa + \theta \Delta W/(bg) + \theta (\Delta L/L)[W/(bg)]\right]
\]

Differentiating \(Y = bLg(\kappa)\) with respect to time and dividing by \(Y\) yields:

\[
\frac{\Delta Y}{Y} = (\Delta b/b) + (\Delta L/L) + \Delta \kappa (g'/g)
\]

\textsuperscript{12} The analysis applies to a generally homogenous production function with modifications and qualifications.
Likewise, differentiating $\kappa = aK/bL$ with respect to time and divide by $\kappa$ yields:

$$\Delta \kappa / \kappa = (\Delta K/K) + (\Delta a/a) - (\Delta L/L) - (\Delta b/b)$$

Then dividing $Y - bLg'(\kappa) = \theta WL$ by $W$ and $Y = bLg$ yields:

$$(1/W) - (\kappa g')/(Wg) = \theta / (bg)$$

Finally, substituting the previous three equations into the prior one and rearranging yields an expression for the labour absorption equation:

$$\Delta L/L = \Delta K/K + \Delta a/a + [(g-g'\kappa)/(g''\kappa^2) - 1](\Delta b/b) + [(g-g'\kappa)/(g''\kappa^2)](\Delta W/W)$$

To interpret this, the mathematical expression for the elasticity of substitution between labour and capital can be written as:\textsuperscript{13}

$$e = \frac{g'g'\kappa^2 - g'g\kappa}{g''g\kappa^2}$$

The equations $Y - bLg'(\kappa) = \theta WL$ and $Y = bLg$ imply that $g'\kappa/g = 1 - \theta WL/Y$, where $\theta WL/Y$ is the tax-adjusted share of income accruing to labour. Define $c = $ the residual—the tax-adjusted share of income accruing to capital, which is equal

\textsuperscript{13} See Bruton and Frank (1977) or Allen (1968).
to \((g'/g)\kappa\). Dividing this expression for \(e\) by \(c = (g'/g)\kappa\) and simplifying yields \(e/c = (g-g')/(g''\kappa^2)\), which allows the simplification of the labour absorption equation as follows:

\[
\Delta L/L = \Delta K/K + \Delta a/a + (e/c - 1)\Delta b/b - (e/c)\Delta W/W
\]

This can be expressed conceptually as:

This expression is the labour absorption equation for the model, and it demonstrates how the rate of job creation is related to not only the rate of investment, but also the net impact of capital- and labour-augmenting productivity growth as well as wages. The \(e/c\) coefficient on wage growth represents a measure of how elastic labour demand is to wages. This coefficient minus one \((e/c - 1)\) is the corresponding measure for the response of labour demand to labour-augmenting productivity growth. The time path of job creation during productivity-driven economic growth depends critically on these coefficients.
References


